KINETIC ENERGY DISSIPATION CAUSED BY THE VORTICAL STRUCTURES IN LINEAR BLADE CASCADE

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Abstract

The dissipation of kinetic energy in a linear blade cascade has been extensively studied from both experimental and numerical perspectives. Nevertheless, the impact of vortices on kinetic energy dissipation in the cascade is often assessed under several assumptions that significantly simplify the situation and may underestimate their influence. The presented paper focuses on the kinetic energy dissipation caused by vortical structures. This investigation is based on a new approach to evaluate individual contributions to the overall kinetic energy dissipation in the cascade. Vortices are identified using the $H$ criterion in this paper. A comparison of this new method with the commonly used approach is presented.

The experimental research was performed on linear blade cascade with pitch-to-chord ratio $t/c = 0.9$ for both constant Mach and Reynolds numbers ($M_{2, is} = 0.4$ and $Re_{2, is} = 2.5 \times 10^5$) and for three different inlet flow angles ($\alpha_1 = -20^\circ; 5^\circ$ and $30^\circ$).

Keywords: Linear blade cascade, vortical structures, kinetic energy dissipation.

1 Introduction

The kinetic energy dissipation (KED) within a linear blade cascade is typically categorized into three main components: the profile, tip clearance, and end-wall KED see e.g. [1]. This classification has led to a widely adopted approach for assessing the KED in the end-wall region. It is based on calculating the difference between the KED in the whole measurement plane at the cascade outlet and the KED at the blade mid-span (profile KED). This relationship can be expressed as:

$$
\zeta_{ew} = \frac{1}{A} \int_A \zeta dA - \frac{1}{2t} \int_y \zeta dy,
$$

(1)

where the on the r.h.s., the first term is the surface integral calculated in the whole outlet flow field, meanwhile the second integral is taken along one line at the blade mid-span. Such an approach is used by several authors, e.g. [2, 3, 4]. This approach relies on the assumption of the superposition of the flow in the end-wall region of the cascade and the idealized 2D flow at the blade mid-span. As demonstrated in this paper, such an approach can significantly underestimate the contribution of end-wall flows to the overall kinetic energy dissipation (KED). Moreover, this evaluation cannot identify the KED caused by the individual vortical structures.

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Area</td>
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<td>$c$</td>
<td>Blade chord</td>
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<td>$H$</td>
<td>$H$ criterion</td>
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<td>$\zeta$</td>
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<td>$M$</td>
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<td>$p$</td>
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<td>$Re$</td>
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<td>$t$</td>
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<td>Components of position vector</td>
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<td>$x, y, z$</td>
<td>Cartesian coordinates</td>
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<td>$\alpha$</td>
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<td>$\varrho$</td>
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<td>$\omega$</td>
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<td>$2, is$</td>
<td>Isentropic at the cascade outlet</td>
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<td>$ew$</td>
<td>End-wall</td>
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<td>$ms$</td>
<td>Mid-span</td>
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<td>$vor$</td>
<td>Vorticity</td>
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2 Aim of the work

Main goal of this work is to examine the KED caused by the vortical structures within a linear blade cascade using experimental data. To achieve this, individual vortices must be identified in the outlet flow field first. Then the contribution of the vortices to the overall KED is calculated for three different inlet flow angles.

3 Experimental apparatus, setup and methods

3.1 Apparatus

The experiments were conducted in the circular low-pressure aerodynamic wind tunnel (WT) at the VZLU laboratory of high-speed aerodynamics Palmovka. The flow in the WT was generated by a twelve-stage radial compressor. Mach number was controlled by adjusting the compressor’s rotational speed. The pressure within the WT was manipulated using a set of vacuum pumps, leading to a decrease in air density and, consequently, variations in the Reynolds number. This wind tunnel design enabled the independent adjustment of both Mach and Reynolds numbers. The schematic representation of the WT is illustrated in Figure 1.

The inlet flow angle was set by a pair of semi-shaped nozzles positioned in front of the WT test section. The cascade was consisted of individual prismatic blades assembled between two acrylic WT windows.

Measurements were conducted using a five-hole pyramid pressure probe at the cascade outlet, positioned 10 mm behind the trailing edges of the blades. The dimensions of this probe and the definitions of the pressure taps are shown in Figure 2.

3.2 Setup

The experiments were conducted on a linear blade cascade consisting of prismatic guiding blades. The pitch-to-chord ratio of the cascade was $t/c = 0.9$. The measurements themselves were carried out under constant Mach ($M_{2,ls} = 0.4$) and Reynolds ($Re_{2,ls} = 2.5 \times 10^5$) isoentropic numbers. The effect of the inlet flow angle was studied for $\alpha = -20^\circ$, $5^\circ$, and $30^\circ$, respectively.
3.3 Methods

Pressures measured by the pyramid five-hole pressure probe were evaluated using the standard procedure. Calibration matrices were employed to correct the measured data, see e.g. [5]. Knowing the true pressures, the velocity vector was calculated at all measured positions in a single plane. Vorticity in the axial direction can then be directly calculated as:

\[
\omega_y = \frac{u^{(i+1),j} - u^{(i-1),j}}{2(z^{(i+1),j} - z^{(i-1),j})} - \frac{u_z^{(i+1)} - u_z^{(i-1)}}{x^{(i+1)} - x^{(i-1)}},
\]

where \(u^{i,j}_\alpha\) represents the component of the velocity vector in the \(\alpha\) direction at the measured point \(i, j\), and \(x\) and \(y\) denote the individual directions. The other vorticity vector components were calculated using Crocco's theorem, e.g. [6]:

\[
\varepsilon_{ijk} u_j \omega_k = \frac{1}{\rho} \frac{\partial p_0}{\partial x_i},
\]

where \(\varepsilon_{ijk}\) is Levi-Civita alternating symbol. The stream-wise vorticity \(\omega_x\) can be calculated as well.

The identification of vortices in a flowing fluid is a complex problem since there is no precise mathematical definition for them to this day. Although attempts to formulate such a definition can be found in available literature, see e.g. [7] or [8], an algorithm for vortex detection cannot be constructed based on these definitions. However, there is a vast number of vortex detection methods in the available literature; for a review, see the paper [9]. The method using helicity density proposed in [10] was adopted for this work. This method is based on a comparison of the \(H\) criteria with a chosen threshold. This criterion is defined as:

\[
H = \frac{H}{|u_i| |\omega_i|} \geq H_{\text{threshold}},
\]

where \(H = u_i \omega_i\) is the helicity density. The impact of the value of \(H_{\text{threshold}}\) on vortex detection was studied in [11], and it was concluded that \(|H_{\text{threshold}}| \geq 0.6\) is sufficient for vortex detection. This value is in agreement with the suggestion given in [12].

Kinetic energy dissipation in the cascade was firstly evaluated based on commonly used data reduction method, e.g. [13]. The results obtained at the cascade mid-span \(\zeta_{ms}\) was then subtracted from the whole flow field:

\[
\zeta_{ew} = \zeta - \zeta_{ms}.
\]

\(\zeta_{ew}\) then represents the KED caused by the end wall flows.

To evaluate the effect of the vortices on the KED in the cascade, a different approach was used. The flow field was divided into three different regions. The first region was the part of the flow field where the flow was considered as two-dimensional. The condition was set that the integral of the KED in this region at individual positions in the \(y\) direction (cascade coordinate system is shown in Figure 3) must have the same values as at the blade mid-span (a small deviation was allowed). The second region was the area near the wall where the flow field did not exhibit a two-dimensional character. In this part of the flow field, the third region identified by the \(H\) criteria as vortices was subtracted.

3.4 Measurement chain and uncertainty

The flow regimes were set according to the measurement of state variables i.e. pressure (static

Figure 3: Definition of the cascade coordinate system.
and stagnation), and the temperature at the cascade inlet and outlet. The barometric pressure was measured by Druck DPI 145 digital pressure transducer with the uncertainty of 0.013% FS. The individual pressures were measured by DRUCK differential pressure transducers with the uncertainty of 0.1% RDG. Temperature and relative humidity were both measured by Sensorika Humistar HTP-1 hygrometer with the precision of $RH \pm 2\%$ and with the precision of $T \pm 0.3\ K$. Mach number was set with uncertainty under 1% and Reynolds number with uncertainty under 2%. Flow velocity was measured with uncertainty under 6% and flow angles with uncertainty under $\pm 1.5^\circ$. The evaluation of the uncertainty was performed for the confidence level of 95% (standard deviation $\pm 2\sigma$).

4 Results and discussions

Figure 4 illustrates the distribution of kinetic energy at the cascade outlet for three inlet flow angles $\alpha_1$. Contours depict regions where vortices were identified by the $H$ criterion. Dashed contours represent vortices rotated in the counterclockwise direction. Vortices with a clockwise direction were identified as the passage vortex, while others were identified as the trailing shed vortex and counterclockwise vortex. With an increasing inlet flow angle, a shift of the vortices and the regions with the highest KED toward the blade mid-span is clearly visible. This shift is caused by the stronger centrifugal forces acting within the cascade due to higher flow turning. This observation is in agreement with theoretical models proposed by, e.g., [14, 15]. It can be observed that the KED maximum corresponds to the center of the vortex in the case of $\alpha_1 = 5^\circ$. For $\alpha_1 = -20^\circ$, the KED maximum was not visible as it was outside the measurement region; therefore, it was impossible to connect the KED maximum with vortices. In the case of $\alpha_1 = 30^\circ$, the KED maximum was observed between two vortices, namely between the passage vortex and the trailing shed vortex. Probe dimensions did not allow the measurements to be performed in close proximity to the wall. The closest point of measurement was 6 mm from the wall, therefore, in the cases of $\alpha_1 = -20^\circ$ and $\alpha_1 = 5^\circ$, all the vortices were not detected.

![Figure 4: Kinetic energy dissipation distribution with identified vortices.](image)
The division of the outlet flow field for calculating the individual KED contributions to the overall KED is illustrated in Figure 4d. Regions with 2D flow, end-wall flow, and individual vortices are highlighted.

(a) Distribution of kinetic energy dissipation along the blade span $z/h$. 
(b) Integral value of dissipation kinetic energy in outlet flow fields for different inlet flow angles.

Figure 5: Integral value of the kinetic energy dissipation obtained by data reduction method (standard procedure of evaluation).

The integral distribution of the KED obtained by the data reduction method in the individual
At the cascade mid-span, the lowest KED was observed for the nominal inlet flow angle $\alpha_1 = 5^\circ$, followed by the underloaded regime $\alpha_1 = -20^\circ$, and the highest value was found for the overloaded regime $\alpha_1 = 30^\circ$. As in the case of local distributions, the highest KED in the end-wall region was detected for $\alpha_1 = 30^\circ$, caused by the stronger end-wall flows. The descending trend in the KED maximum with decreasing inlet flow angle is evident.

Applying equation (5) to the data, Figure 5b was obtained. The integral KED in the flow field was calculated again using the data reduction method. The increasing trend was mostly caused by the end-wall KED, as can be deduced from the figure. The variation of the mid-span KED was not highly affected by the parameter $\alpha_1$ in this case.

In the case of evaluation based on vortex identification by the $H$ criterion, the results were different, see Figure 6a. The individual contributions to the KED were evaluated using the formula:

$$\zeta_i = \frac{1}{A} \int_A \zeta \left( \frac{1}{c} \frac{x}{h} \right) dA_i,$$

where $\zeta \left( \frac{1}{c} \frac{x}{h} \right)$ is the local KED, $A$ is the surface of the whole outlet flow field, and $A_i$ are the surfaces of the regions where the individual contributions of the KED $\zeta_i$ were calculated. Here, $i$ can be ms for mid-span, ew for end-wall, and vor stands for vortices. The sum of all these contributions again gave the whole integral KED in the flow field, as in the previous case. However, as mentioned above, the other results were different. The contribution of end-wall flows to the overall KED was twice the value of the original method, and the contribution of the mid-span KED was significantly lower. More specifically, $\zeta_{ew}$ increased from approximately 0.002 to 0.006 for the case of $\alpha_1 = -20^\circ$ and from 0.01 to 0.022 for $\alpha_1 = 30^\circ$. Moreover, the mid-span KED for $\alpha_1 = -20^\circ$ was higher compared to other studied cases. It can be speculated that if the probe was able to measure closest to the wall, the mid-span values might not have been constant over the whole range of $\alpha_1$.

The contribution of the vortices to the overall KED increased with $\alpha_1$, which is in agreement with the theory of secondary flow. With higher flow turning in the cascade, stronger vortex structures are expected, leading to even higher KED.

Figure 6b shows the comparison of the used methods based on the relation:

$$\frac{|\Delta \zeta_i|}{\zeta} = \frac{|\zeta_{old,i} - \zeta_{H,i}|}{\zeta},$$

where $i$ stands for mid-span flow if $i = ms$ and for end-wall flow if $i = ew$. The absolute value ensures a positive result in the case of subtracting the end-wall KEDs. Here, normalized differences
in the KED between the different approaches are plotted again as a function of $\alpha_1$. This difference rose with the inlet flow angle from approximately 10 to 30% of the overall KED in the case of mid-span and from approximately 15 up to 35% in the case of the end-wall.

5 Note on the flow field division into different regions

The Navier-Stokes equations, which model fluid flow in a perfect approximation, constitute sets of non-linear partial differential equations. In non-linear systems, the superposition of phenomena is generally not correct. Such an approach can be used in the case of linear systems. The commonly used superposition approach is, of course, easy to use and, therefore, is usually employed, providing useful insights into the problem, especially in cases where the flow at the blade mid-span is truly two-dimensional.

However, as mentioned above, in the case of flow separation at the blade mid-span, the end-wall effects are completely overwhelmed by this separation, even though the vortex structures are clearly visible. Another reason for employing the approach presented in this paper is the capability of KED assignment to individual vortices rather than obtaining only information about the end-wall flow as a whole. This method can be used in the future to study the effects of individual vortical structures on cascade performance. Presently, many works focus on flow control with the goal of suppressing end-wall flow effects, see e.g [16]. In such cases, this presented approach can provide a useful tool to evaluate the impact of flow control on individual vortices.

6 Conclusion

Investigation of the effect of end-wall flows and vortical structure on the overall KED in the linear blade cascade was the topic of this paper. The commonly used method (based on data reduction methods), which assumes the superposition of the 2D flow at the blade mid-span with end-wall flows, was compared with a new approach based on the identification of individual vortices via the $H$ criterion. It was shown that the method used to evaluate individual KED contributions to the overall KED significantly affects the results. The method based on the $H$ criterion shows that the effect of end-wall flows can be as much as twice that of the previously used approach. This is caused by the different division of the outlet flow field. The method presented here, in the author’s opinion, better describes the situation, as the mentioned assumption about the flow superposition is not correct, as mentioned in section 5. Moreover, in [2], flow separation on the blade suction surface occurred. This separation then overwhelmed the effects of the end-wall flows completely, although the vortices and end-wall structures were clearly visible.

There are two additional tasks that need to be addressed in future work for a better understanding of the end-wall flows:

1. The experiments that include the regions closer to the wall need to be performed to investigate this area and reveal all of the vortices. A special five-hole pressure probe has already been manufactured by the author for this purpose and will hopefully be used soon.

2. The division of the outlet flow field into mid-span and end-wall flows was performed manually in this work. It would be useful to find some criterion for this division that can be used in the evaluation procedure automatically.

References


