NEW MATHEMATICAL MODEL OF DIELECTRIC BARRIER DISCHARGE PLASMA ACTUATOR IN AIR

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Abstract
The goal of this study is to provide a thorough numerical and analytical physics-based model that can qualitatively and quantitatively characterize the processes of dielectric barrier discharge plasma actuator functioning. The drift-diffusion approach to describe the spatial-temporal structure of the dielectric barrier discharge in air at atmospheric pressure, including kinetic phenomena and plasma-chemical reactions, was chosen as the basic one. Electronically excited and metastable states of nitrogen molecules and oxygen, oxygen atoms, electrons, as well as positive and negative ions, a total of 14 particles and 97 plasma-chemical reactions, including surface processes were considered in this paper. Chemical reactions include processes of dissociation, ionization of molecules by electron impact from the ground state, stepwise, associative and photoionization, excitation of molecules, ionization of excited (metastable) molecules, attachment and reattachment of electrons, recombination of electrons and positive ions, chemical transformations of neutral atoms, molecules and ions, and also processes of secondary emission of electrons from an open electrode and a dielectric surface. The temperature, mobility and diffusion of electrons, as well as the coefficients of some chemical reactions (ionization, excitation, attachment) depend on the electric field strength and are calculated using the BOLSIG+ solver, which is based on solving the Boltzmann equation using the electron velocity and energy distribution function. Test calculations of the plasma flow generation and development near the flat plate are carried out.

Keywords: dielectric barrier discharge, plasma actuator, plasma dynamics

1 Introduction
The task of air flow separation control is topical for development of aviation, rocket and space technology, engines and turbine engineering, wind power [1-5]. Existing methods of air flow separation control are energy-consuming and require changes in the design of the streamlined body (surface cooling, perforation, installation of interceptors, etc.). In this regard, the development of effective and low-cost methods for preventing flow separation is one of the most important areas of fundamental and applied aerodynamics [6-8]. The use of plasma actuators (PA) on the basis of the dielectric barrier discharge (DBD) is among the modern and promising ways to change structures [1-8]. Unlike classical methods, the use of DBD is less energy consuming and does not require constructive changes (surface punching, installation of spoilers or additional moving parts).

These plasma actuators do not allow creating additional obstacles. They ionize the external flow and due to the Lorentz force arising, they allow to achieve the desired flow structure [9, 10]. Experimental data of foreign authors confirm the effectiveness of this method for flow separation control [4, 5, 11-13].

Existing mathematical models of the dynamics of a partially ionized flow rely on empirical constants and are suitable only for a limited class of flows [1-4, 6, 7]. The development of an adequate model for describing the processes studied in a wide range of determining parameters is an actual and not yet solved problem in this field of knowledge. A developed mathematical model should include the main physical and chemical processes taking place - convection, turbulence, viscous-inviscid interaction, flow ionization, energy and electron transfer, electrochemical transformations, photoionization [4, 5, 11-13]. A separate problem is the development of specialized software and methodological support for the numerical reconstruction of the studied processes [14, 15]. Modern commercial software packages of computational
aerodynamics quite well reproduce stationary incompressible turbulent flows. However, they do not take into account most of the phenomena necessary for modeling the dynamics of a partially ionized flow such as the emergence and transfer of electrons and ions, photoionization, electrochemical processes. In addition, the need to model non-stationary processes imposes additional restrictions on the numerical algorithms.

2 Statement of the problem of the study of electrodynamics, plasma dynamics and chemical kinetics

The plasma actuator based on the DBR consists of two electrodes located asymmetrically, which are separated by a dielectric, as shown in Fig. 1. One of the electrodes is open and in contact with air, and the other is completely immersed in a dielectric material (glass, quartz, polymers). The electrodes are located on the aerodynamic surface along the span of the streamlined body.

![Figure 1: Scheme of the plasma actuator [5]](image)

Based on the results of previous studies on the methods of mathematical description of low-temperature nonequilibrium ideal plasma [16-18], the drift-diffusion approach to describe the spatial-temporal structure of the dielectric barrier discharge in air at atmospheric pressure, including kinetic phenomena and plasma-chemical reactions, was chosen as the basic one.

2.1 Kinetic scheme of the dielectric barrier discharge plasma

The working gas is air with a fixed proportion of nitrogen $N_2/N = 0.78$ and oxygen $O_2/N = 0.22$ at normal pressure at sea level $p = 101325$ N/m$^2$ (1 atm.). The air temperature is assumed to be constant and equal $T = 300 K$. The total number of nitrogen and oxygen molecules per unit volume is $N = 2.447 \times 10^{25}$ 1/m$^3$. Electronically excited and metastable (*) states of nitrogen molecules $N_2^*(A^1 \Sigma_u^+)$, $N_2^*(B^1 \Pi_g)$, $N_2^*(a^3 \Sigma_u^+)$, $N_2^*(C^3 \Pi_u)$ and oxygen $O_2^*(a^3 \Delta_g)$, $O_2^*(b^3 \Sigma_u^+)$, oxygen atoms $O$, electrons $e$, as well as positive $N_2^+$, $N_2^*$, $O_2^+$, $O_2^*$, and negative ions $O^-$, $O_2^-$, a total of 14 particles and 97 plasma-chemical reactions, including surface processes were considered at this paper.

Chemical reactions include processes of dissociation, ionization of molecules by electron impact from the ground state, stepwise, associative and photoionization, excitation of molecules, ionization of excited (metastable) molecules, attachment and reattachment of electrons, recombination of electrons and positive ions, chemical transformations of neutral atoms, molecules and ions, and also processes of secondary emission of electrons from an open electrode and a dielectric surface.

Since the dielectric barrier discharge plasma is non-equilibrium, the ion temperature is equal to the air temperature. The temperature, mobility and diffusion of electrons, as well as the coefficients of some chemical reactions (ionization, excitation, attachment) depend on the electric field strength and are calculated using the BOLSIG+ solver [19], which is based on solving the Boltzmann equation using the electron velocity and energy distribution function.

2.2 Equations of plasma electrodynamics

In the general case, plasma can be described by four Maxwell equations in the form

$$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t, \quad \nabla \times \mathbf{H} = \mathbf{j} + \partial \mathbf{D}/\partial t, \quad (1)$$
where $\mathbf{H}$ – the magnetic field strength, $\mathbf{B}$ – the magnetic induction, $\mathbf{E}$ – the electric field strength, $\mathbf{D}$ – the electric induction, $\mathbf{j}$ – the density of electric current, $\rho_\pm = \varepsilon (n_+ - n_-)$ – the density of the resulting volume charge, $\varepsilon$ – the elementary charge, $n_+, n_-$ – the volume density of positive and negative particles. Equations (1) represent the Gauss law, the Gauss law for the magnetic field, the Faraday law and the Ampere-Maxwell law, respectively.

The Faraday law and the Gauss law for magnetic induction are executed identically, if the electric and magnetic fields are expressed in terms of scalar and vector potentials

$$\mathbf{E} = -\nabla \varphi - \partial \mathbf{A}/\partial t, \quad \mathbf{B} = \nabla \times \mathbf{A}.$$ 

The vector of electrical induction $\mathbf{D}$ is associated with the vector of electric field strength $\mathbf{E}$ through the absolute dielectric constant $\varepsilon = \varepsilon_r \varepsilon_0$ and is equal to $\mathbf{D} = \varepsilon \mathbf{E}$, where $\varepsilon_r$ is the relative dielectric constant of the medium, $\varepsilon_0$ is the electric constant.

Since the speeds of motion of charged particles in plasma are much lower than relativistic velocities and there are no external sources of a magnetic field, then the magnetic field strength $\mathbf{H}$ and magnetic induction $\mathbf{B}$ are assumed to be zero. In addition, the time derivative of the magnetic induction $\partial \mathbf{B}/\partial t$ is zero. Then $\mathbf{E} = -\nabla \varphi$ and the Gauss law taking into account the surface charge will take the form

$$V \cdot (\varepsilon \nabla \varphi) = -\rho / \varepsilon_0 - \sigma \delta / \varepsilon_0,$$

where $\delta$ – the Dirac delta function, $\sigma$ – the total surface density of the electric charge. Equation (2) is the Poisson equation for the electric field.

The density of the resulting charge at any point of the plasma is defined as the difference between the density of the positive and negative charge. Then you can write

$$V \cdot (\varepsilon \nabla \varphi) = -\varepsilon (n_+ + n_{i+} + n_{e+} - n_{i-} - n_{e-} - n_0)/\varepsilon_0 - (\sigma_+ - \sigma_-) \delta / \varepsilon_0,$$

where $\sigma_+, \sigma_-$ – the surface density of positive and negative charges, $n_+, n_{i+}, n_{e+}, n_{i-}, n_{e-}, n_0$ – the bulk density of electrons, as well as positive and negative ions of nitrogen and oxygen.

### 2.3 The equations of the plasma particles dynamics in the drift-diffusion approximation

Based on the kinetic scheme of the dielectric barrier discharge, it is possible to make the equations of dynamics for each kind of particles. The system of equations for the dynamics of plasma particles in a drift-diffusion formulation can be written in Cartesian form

$$\frac{\partial \mathbf{n}}{\partial t} - \frac{\partial}{\partial x} \left( \mathbf{u} \cdot \frac{\partial \mathbf{E}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mathbf{u} \cdot \frac{\partial \mathbf{E}}{\partial y} \right) - \frac{\partial}{\partial x} \left( \mathbf{D} \cdot \frac{\partial \mathbf{E}}{\partial x} \right) - \frac{\partial}{\partial y} \left( \mathbf{D} \cdot \frac{\partial \mathbf{E}}{\partial y} \right) = \mathbf{S},$$

where $\mathbf{n}$ – the vector of unknown variables for the bulk density of particles, $\mathbf{u}$ and $\mathbf{D}$ – the vector coefficients of mobility and diffusion of particles, $\mathbf{S}$ – the vector of source terms

\[
\mathbf{n} = \begin{bmatrix} n_{i+}, n_{e+}, n_{i+}(c^{+})^{n_{i+}}, n_{e+}(c^{+})^{n_{e+}}, n_{i+}(c^{0})^{n_{i+}}, n_{e+}(c^{0})^{n_{e+}}, n_{e+}, n_{i+}, n_{e+} \end{bmatrix}^T,
\]

\[
\mathbf{u} = \begin{bmatrix} \mu_{i+}, \mu_{e+}, 0, 0, 0, \mu_{i+}^{\text{aq}}, \mu_{e+}^{\text{aq}}, -\mu_{e+}^{\text{aq}}, -\mu_{i+}^{\text{aq}}, 0, 0, 0, -\mu_e \end{bmatrix}^T,
\]

\[
\mathbf{D} = \begin{bmatrix} D_{i+}, D_{e+}, 0, 0, 0, D_{i+}^{\text{aq}}, D_{e+}^{\text{aq}}, D_{i+}^{\text{aq}}, D_{e+}^{\text{aq}}, 0, 0, 0, D_e \end{bmatrix}^T,
\]

\[
\mathbf{S} = \begin{bmatrix} S_{i+}, S_{i+}(c^{+})^{n_{i+}}, S_{e+}(c^{+})^{n_{e+}}, S_{i+}(c^{0})^{n_{i+}}, S_{e+}(c^{0})^{n_{e+}}, S_{i+}, S_{i+}^{\text{aq}}, S_{e+}, S_{e+}^{\text{aq}}, S_{i+}^{\text{aq}}, S_{e+}^{\text{aq}} \end{bmatrix}^T.
\]

Here, the product of the form $\mathbf{nm}$ refers to the vector $\left[ \mu n_{i+}, \mu n_{e+}, \ldots, \mu n_i \right]^T$.

The components of the vector of source terms in expression (3) are formed on the basis of the kinetic scheme of the dielectric barrier discharge and are responsible for the sources and sinks of a certain sort of particles. The principle of formation of source members, the description of bulk, surface chemical reactions and their coefficients, as well as the values of transfer coefficients (mobility and diffusion) of positive and negative ions are given in [16, 18].

The speed of movement of neutral and excited particles can be neglected, since it is comparable to the speed of movement of a continuous medium and is 2-3 orders of magnitude less than the speed of
movement of charged particles. In addition, the rate of chemical processes involving neutral and excited particles is much higher than the diffusion processes.

2.4 Balance equations of surface density of positive and negative charge

The processes on the surface of the dielectric play an essential role in the operation of the plasma actuator. Thus, the interaction of charged particles with a dielectric surface leads to the accumulation of electric charge on the surface of the dielectric. The balance equation of the surface density of positive and negative charge is determined by the following expressions

\[ \frac{\partial \sigma_+}{\partial t} = -e(1 + \gamma_{\text{ion}}) \Gamma_{i+} - \alpha_{\text{rec}} \sigma_+ / e, \quad \frac{\partial \sigma_-}{\partial t} = -e \Gamma_{i-} - \alpha_{\text{rec}} \sigma_- / e, \]

where \( \Gamma_{i+}, \Gamma_{i-}, \Gamma_e \) – the flux of positive, negative ions and electrons normal to the surface, which is determined by the type of boundary conditions (Table 1), \( \sigma_+, \sigma_- \) – is the surface density of positive and negative charges, \( \alpha_{\text{rec}} \) – the coefficient of surface recombination, \( \gamma_{\text{ion}} = 0.005 \) – the coefficient of ion-electron emission from a dielectric. The surface recombination coefficient \( \alpha_{\text{rec}} \), is determined by the surface diffusion of electrons

\[ \alpha_{\text{rec}} = d_r \sqrt{\pi k_B T_m / m_e}, \]

where \( d_r = 10^{-5} m \) – the recombination radius, \( T_m \) – the temperature of the dielectric surface.

2.4 Initial and boundary conditions for the initial system of equations

**Poisson equation for electric potential.** As the initial conditions for the Poisson equation, the zero distribution of the electric potential in the region was specified. Equation (2) is solved for the electric potential using the applied voltage to the electrodes as the boundary condition, as well as the corresponding values of the relative permittivity for air and dielectric. The alternating voltage applied to the open electrode is given by

\[ \phi(t) = \phi^{\text{max}} \sin(2\pi \omega t), \]

where \( \omega \) – frequency, \( \phi^{\text{max}} \) – amplitude of oscillations. A zero potential is applied to the insulated electrode. At the outer boundaries, the Neumann condition is imposed \( \partial \phi / \partial \theta_n = 0 \).

**The equations of the plasma particles dynamics.** As the initial conditions for the equation of the dynamics of charged plasma particles, the background concentration of ions and electrons in the air was set (\( n_i = 10^9 \, 1/m^3, n_e = 10^9 \, 1/m^3 \)). The boundary conditions for the equations of the dynamics of charged particles on a solid surface are given in Table 1, where \( V_{sd} \) – the thermal velocity of the particles, \( \gamma_{\text{sr}} \) – the coefficient of ion-electron emission from the copper anode, which depends on the electric field intensity. At the outer boundaries, the Neumann condition is set \( \partial n / \partial \theta_n = 0 \). The thermal velocity of the particles is determined by the formula

\[ V_{sd} = \sqrt{8 k_B T_e / \pi m_e}, \]

where \( m_e, T_e \) – the mass and temperature of ions and electrons, \( k_B \) – the Boltzmann constant.

Table 1: Boundary conditions for the charged particles dynamics equations

<table>
<thead>
<tr>
<th>( E_i )</th>
<th>( \Gamma_{i+} )</th>
<th>( \Gamma_{i-} )</th>
<th>( \Gamma_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &gt; 0 )</td>
<td>( -1/4n_i V_{sd}^i )</td>
<td>( -\mu_e n_e - 1/4n_i V_{sd}^i )</td>
<td>( -\mu_e n_e - 1/4n_i V_{sd}^i )</td>
</tr>
<tr>
<td>( \leq 0 )</td>
<td>( \mu_e n_e - 1/4n_i V_{sd}^i )</td>
<td>( -1/4n_i V_{sd}^i )</td>
<td>( -\gamma_{\text{sr}} )</td>
</tr>
</tbody>
</table>

3 Numerical method

3.1 The system of equations for the plasma particles dynamics and electrodynamics in a curvilinear coordinate system

**Unsteady formulation.** To simulate a dielectric barrier discharge, it is necessary to jointly solve the system of equations of plasma particle dynamics with the Poisson equation for the electric field. When solving a non-stationary problem of plasma dynamics, heterogeneous equations are considered. To match them, we introduce the pseudo-time \( \tau \) for each equation.
where \( r_o \) and \( r_p \) – pseudo-time for the equations of the plasma particles dynamics and electric potential, 
\[ n_+ = n_{i+1}^+ + n_{0+}^+ + n_{e+}^+ + n_{-}^-, \quad n_- = n_{i-1}^- + n_{0-}^- + n_{e-}^- \] – the bulk density of positive and negative particles. In the future, for the convenience of deriving the necessary equations, the last term in (4) will be omitted.

The equations of the plasma particles dynamics. We will consider the equation for the particle density in the drift-diffusion approximation in a curvilinear coordinate system in the form

\[ \frac{\partial n}{\partial t} + \frac{\partial \vec{n}}{\partial t} + \frac{\partial \vec{H}}{\partial \xi} - \frac{\partial \vec{H}_n}{\partial \eta} - \frac{\partial \vec{D}}{\partial \xi} - \frac{\partial \vec{D}_n}{\partial \eta} = \vec{S}, \]  

(5)

where \( \vec{n} = n/J, \vec{S} = S/J, J \) – the Jacobian of coordinate transformations, \( \xi, \eta, \zeta, \eta \) – the metric coefficients,

\[ \vec{H}_n = \frac{m}{J} \left( \frac{\partial \phi}{\partial \xi} + \xi \eta \frac{\partial \phi}{\partial \eta} \right), \quad \vec{H}_n = \frac{m}{J} \left( \frac{\partial \phi}{\partial \xi} + \eta \frac{\partial \phi}{\partial \eta} \right), \]

(6)

\[ \vec{D}_n = \frac{D}{J} \left( \frac{\partial \phi}{\partial \xi} + \xi \eta \frac{\partial \phi}{\partial \eta} \right), \quad \vec{D}_n = \frac{D}{J} \left( \frac{\partial \phi}{\partial \xi} + \eta \frac{\partial \phi}{\partial \eta} \right). \]

(7)

The terms \( \partial \vec{H}_n/\partial \xi, \partial \vec{H}_n/\partial \eta \) are responsible for the advection (drift) of charged particles. A formal mathematical approximation of expressions (6) using symmetric finite-difference relations (as for diffusion terms (7)) leads to a loss of the physical meaning of this operator as advection of charged particles. To preserve the physical meaning of advection, an asymmetric finite-volume approximation is introduced for \( \phi \), taking into account advection by \( n \) type

\[ \langle n_i \rangle_{\xi \eta} = \begin{cases} n_i + \Psi_{i,1/2} \left( n_{i+1} - n_i, n_{i-1} - n_i \right), & -(\mu \nu \phi)_{i+1/2} \geq 0, \\ n_i - \Psi_{i,1/2} \left( n_{i-1} - n_i, n_{i+1} - n_i \right), & -(\mu \nu \phi)_{i+1/2} < 0. \end{cases} \]

(8)

where \( \Psi_{i,1/2} \) – the MinMod flow limiter of the second order of accuracy.

Equation for electric potential. The Poisson equation for the electric potential in a curvilinear coordinate system will take the following form

\[ \frac{\partial \phi}{\partial t} + \frac{\partial \hat{\phi}}{\partial \xi} + \frac{\partial \hat{\phi}}{\partial \eta} = \frac{\hat{\rho}_p}{\epsilon_0}, \]

(9)

where \( \phi = \phi/J, \hat{\rho}_p = \rho_p/J \).

\[ \hat{\phi}_i = \frac{\epsilon_i}{J} \left( \xi \eta + \xi \xi \right) \left( \frac{\partial \phi}{\partial \xi} + \xi \eta \frac{\partial \phi}{\partial \eta} \right), \quad \hat{\phi}_\eta = \frac{\epsilon_\eta}{J} \left( \xi \eta + \eta \eta \right) \left( \frac{\partial \phi}{\partial \xi} + \eta \frac{\partial \phi}{\partial \eta} \right). \]

(10)

Approximation of the second derivatives for \( \phi \) (10) is performed by finite volume relations.

### 3.2 Implicit method for the equations of the plasma particles dynamics and electrodynamics

The equations of the plasma particles dynamics. Consider equation (5) on a new time layer

\[ \frac{\partial \hat{n}_n}{\partial \tau_n} + \frac{\partial \hat{n}_n}{\partial t} + \frac{\partial \hat{H}_n}{\partial \xi} - \frac{\partial \hat{H}_n}{\partial \eta} - \frac{\partial \hat{D}_n}{\partial \xi} - \frac{\partial \hat{D}_n}{\partial \eta} = \hat{S}_n, \]

or

\[ \frac{(\hat{n}_n^{i+1} - \hat{n}_n^n)}{\Delta \tau_n} = \hat{R}_n^{i+1,n} + \hat{S}_n^{i+1,n} - \left( 1.5 \hat{n}_n^{i+1} - 2 \hat{n}_n^n + 0.5 \hat{n}_n^{-1} \right) / \Delta t, \]

(11)

where

\[ \hat{R}_n^{i+1,n} = \frac{\partial \hat{H}_n^{i+1,n}}{\partial \xi} + \frac{\partial \hat{H}_n^{i+1,n}}{\partial \eta} + \frac{\partial \hat{D}_n^{i+1,n}}{\partial \xi} + \frac{\partial \hat{D}_n^{i+1,n}}{\partial \eta}. \]

(12)
The algorithm used is based on a three-layer implicit scheme with subiteracy in pseudo-time \( \tau_n \), second-order accuracy in physical time \( \tau \). Linearize the residual \( \dot{\mathbf{R}} \), source term \( \dot{\mathbf{S}} \), and write equation (11) in delta form

\[
\left[ \left( 1/(J \Delta \tau_n) + 1.5/(J \Delta t) \right) \mathbf{E}_{i+1}\tau_n - \left( \frac{\partial \mathbf{R}}{\partial \mathbf{n}} \right)^{n+1,m} + \left( \frac{\partial \dot{\mathbf{S}}}{\partial \mathbf{n}} \right)^{n+1,m} \right] \Delta \mathbf{n}^{n+1,m} = \mathbf{0},
\]

where \( \Delta \mathbf{n}^{n+1,m} = \mathbf{n}^{n+1,m} - \mathbf{n}^{n,m} \), \( \mathbf{E}_{i+1}\tau_n \) the identity matrix. Linearization of equation (19) is performed by pseudo-time.

The equation of electric potential taking into account the equations for the charged particles density. When solving the Poisson equation for the electric potential together with the equations of the dynamics of plasma particles, an important role is played by the integration step in time. Explicit matching of these equations imposes a limit on the time step of the form \( \Delta t \leq \Delta t_{\text{Maxwell}} \), where \( \Delta t_{\text{Maxwell}} = \frac{\varepsilon_0}{\sum \varepsilon \mu n} \).

Maxwell time (the relaxation time of the space charge) is the characteristic time for charged particles to establish an equilibrium state under the action of an alternating electric field, which they also change.

The relationship of the Poisson equation with the equations of the dynamics of charged particles consists in calculating the transport of particles in the total electric field, which consists of the electric field generated by the same charged particles, and the external electric field.

Consider equation (9) at the new time layer \( n+1 \) for subiteration \( m+1 \)

\[
\frac{\partial \phi^{n+1,m+1}}{\partial \tau_n} + \frac{\partial \phi^{n+1,m+1}}{\partial \xi} + \frac{\partial \phi^{n+1,m+1}}{\partial \eta} = -\frac{e}{\varepsilon_0} \left( \hat{n}_e^{n+1,m+1} - \hat{n}_e^{n+1,m} \right).
\]

From the Taylor expansion in pseudo-time \( \tau \) for values \( n_e \) and \( n \), we get

\[
\hat{n}_e^{n+1,m+1} = \hat{n}_e^{n+1,m} + \Delta \tau_n \left( \frac{\partial \mathbf{n}_e}{\partial \mathbf{n}} \right)^{n+1,m+1} + O(\Delta \tau_n^3).
\]

Substituting (14) into (13), we get

\[
\frac{\partial \phi^{n+1,m+1}}{\partial \tau_n} + \frac{\partial \phi^{n+1,m+1}}{\partial \xi} + \frac{\partial \phi^{n+1,m+1}}{\partial \eta} = -\frac{e}{\varepsilon_0} \left( \hat{n}_e^{n+1,m+1} - \hat{n}_e^{n+1,m} \right)^{n+1,m+1} - \frac{e \Delta \tau_n}{\varepsilon_0} \left( \frac{\partial \mathbf{n}_e}{\partial \mathbf{n}} \right)^{n+1,m+1} + O(\Delta \tau_n^3).
\]

From equation (5) we have

\[
\left( \frac{\partial \mathbf{n}_e}{\partial \mathbf{n}} \right)^{n+1,m+1} = \left( -\hat{n}_e \frac{\partial \hat{H}_e}{\partial t} + \frac{\partial \hat{H}_e}{\partial \xi} + \frac{\partial \hat{H}_e}{\partial \eta} + \frac{\partial \hat{D}_e}{\partial \xi} + \frac{\partial \hat{D}_e}{\partial \eta} + \hat{S} \right)^{n+1,m+1},
\]

where \( L_e = L_{\hat{H}_e} + I_{\hat{D}_e} \), \( I_{\hat{D}_e} = \hat{n}_e \hat{H}_e \), \( L_{\hat{D}_e} = \hat{n}_e \hat{H}_e \).

Substituting (15) into (16), we get

\[
\frac{\partial \phi^{n+1,m+1}}{\partial \tau_n} + \frac{\partial \phi^{n+1,m+1}}{\partial \xi} + \frac{\partial \phi^{n+1,m+1}}{\partial \eta} + e \Delta \tau_n \left( \frac{\partial \hat{H}_e}{\partial \xi} + \frac{\partial \hat{H}_e}{\partial \eta} \right)^{n+1,m+1} + \frac{\partial \hat{D}_e}{\partial \eta} = -\frac{e}{\varepsilon_0} \left( \hat{n}_e - \hat{n} \right)^{n+1,m+1} - \frac{e \Delta \tau_n}{\varepsilon_0} \left( \frac{\partial \mathbf{n}_e}{\partial \mathbf{n}} \right)^{n+1,m+1} + O(\Delta \tau_n^3).
\]

If the numerical algorithm first solves the equation with respect to \( \phi \), and then the equations for the density of charged particles \( n_e \) and \( n \), then in relation (17) all coefficients are taken from the previous subiteration \( m \). Numerical approximation of the derivatives \( \partial \hat{n}_e / \partial t \) and \( \partial \hat{n}_e / \partial t \) in (16) can be performed with the required order of accuracy in \( \Delta t \). The use of Taylor expansion does not impose restrictions on the type of scheme (explicit or implicit) for equations of the form (5).

As a result, we have the Poisson equation for the electric potential \( \phi \) on the time layer \( n+1,m+1 \) taking into account the bulk density of charged particles \( n_e \) and \( n \) on the same time layer in the form

\[
\left( \frac{\partial \phi^{n+1,m+1}}{\partial \tau_n} - \frac{\partial \phi^{n+1,m}}{\partial \tau_n} \right) / \Delta \tau_n = -\hat{R}_\phi^{n+1,m+1} + \hat{S}_\phi^{n+1,m+1},
\]

\[
\hat{R}_\phi^{n+1,m+1} = \frac{\partial \phi^{n+1,m+1}}{\partial \xi} + \frac{\partial \phi^{n+1,m+1}}{\partial \eta} + e \Delta \tau_n \left( \frac{\partial \hat{H}_e}{\partial \xi} + \frac{\partial \hat{H}_e}{\partial \eta} \right)^{n+1,m+1} + \frac{\partial \hat{D}_e}{\partial \eta}.
\]
Electrodes are strips of copper. The length of the open electrode is 5 mm, and the insulated one is 25 mm. A voltage with amplitude of $\varphi = 7$ kV [11] and $\varphi = 12$ kV [12], with a frequency of 5 kHz and 200 Hz, respectively, was applied to the upper electrode. A quarter of the applied voltage oscillation period is considered to demonstrate the capabilities of the developed new mathematical model. In the experiment, the dielectric surface consisted of small segments, which made it possible to measure the voltage distribution over the dielectric surface. A variable time integration step $(\Delta t = 10^{-7} + 10^{-8}s)$ is used to adequately describe the origin, development, and passage of the streamer. The origin of the coordinates coincides with the right edge of the open electrode.

During the development of the streamer, the surface of the dielectric is charged, because the resulting voltage charge, when moving in the electric field, encounters an obstacle in the form of the surface of the dielectric and settles on it. The positive charge on the surface of the dielectric is mainly provided by nitrogen $N_{2}^{+}$ and oxygen $O_{2}^{+}$ ions. The distribution of the electric potential on the surface of the dielectric was obtained at the maximum voltage applied to the electrodes, 7 and 12 kV (Fig. 2). A sharp drop in the voltage on the dielectric is due to the length of the streamer spread and, as a result, a drop in the surface charge density. Thus, at the maximum values of the applied voltage of 7 and 12 kV, the streamer propagation length is $L = 0.01$ m and $L = 0.015$ m, respectively. The results of the numerical simulation show a satisfactory agreement with the experimental data.
5 Conclusions

A new mathematical model of a dielectric barrier discharge during the operation of the plasma actuator in air at atmospheric pressure is developed. It is describe of the space-time structure including unsteady electrodynamic processes, kinetic phenomena and plasma-chemical reactions. A new mutually agreed system of initial equations is proposed, consisting of the equation for electric potential and 14 equations of charged plasma particles dynamics. It is written in an arbitrary curvilinear coordinate system and using a different pseudo-time scale in various equations. A numerical-analytical modification of the Poisson equation for the electric potential $\varphi$ in a curvilinear coordinate system was developed for direct extraction of operators by $\varphi$, instead of indirect influence through the density of charged particles $n_{\pm}$ in the source term, using upwind approximation of the charged particles density in the second derivatives for electric potential. A common implicit numerical algorithm was implemented to effectively solve the inhomogeneous system of initial equations. Test calculations of the plasma flow generation and development near the flat plate are carried out.

References


