MODELING OF THE SURFACE FILTRATION OF AN AQUEOUS SUSPENSION

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Abstract
Filtration of an aqueous suspension with a two-fractional impurity through a layer of deposit formed from larger particles is considered using analytical methods. The absorption of smaller particles by the dynamic layer and the resulting decrease in the filtration rate, also increase in pressure losses are analyzed in two filtration modes - with constant rate and head drop. A technological analysis was performed on a number of examples and the possibility of controlling the start time of the protective action of the specified layer due to the regulation of its formation was shown.

Keywords: surface filtration, modeling, deposit, calculation

1 Introduction
Surface filtration is successfully used along with traditional volumetric filtration in the practice of water treatment. The disperse impurity is retained in the entire volume of the filtering material of rapid filters [1-3]. At the same time, they are operated for a long time due to regular backwashing and thus restoring the previous adsorption capacity. If the surface filtration is implemented, then the surface (dynamic) layer is gradually formed either on a special holding surface with fine perforations or on the surface of the packed bed and is stored for a filter run [4, 5]. The surface filtration is applied independently in the first case, and, as an addition to the volumetric filtration in the second case. The surface filtration is practically organized mainly due to the sieve effect, namely, depending on the ratio between the sizes of the effective pores in the packed bed and larger suspended particles [6, 7].

In the practice of filtration and in membrane technologies, a dynamic layer is often formed intentionally in order to intensify the volumetric filtration or to take full advantage of the surface filtration [8, 9]. The advantages of the filtration using the dynamic layer include, first of all, the slow growth of head losses and the absence of the need for systematic backwashing. Although it is impossible not to mention its significant drawback - low protective ability at the initial stage without the bed. To manage the formation of a dynamic layer with desired properties, it is enough to select the appropriate dispersed substance (sorbent), dose the amount being added into the initial water and the hydraulic loading. The undetachable filtration was studied by analytical methods sequentially in two modes: at a constant filtration rate \( V = \text{const} \) and a constant head drop at the boundaries of the dynamic layer \( \Delta h = \text{const} \) [10-12]. Particles of both types do not interact with each other in a suspended state. The deposit in the pores of the dynamic layer is formed exclusively from small particles and a relatively large amount of bound water.

The basic mathematical model of the surface filtration includes interconnected clarification and hydraulic compartments. The first compartment is based on the equations of conservation and kinetics of the removal of the small suspended particles. The second compartment includes the equation of motion and expressions characterizing the permeability of the contaminated porous medium and the deposit composition. The statement of the mathematical problems to be solved is formulated in a dimensionless form as follows

\[
\nabla \left( f \right) \frac{\partial C}{\partial \tau} - \frac{\partial S}{\partial t} = 0; \quad (1)
\]

\[
\frac{\partial S}{\partial t} = \mathcal{K} \left( S \right) C; \quad (2)
\]

\[
\nabla \left( f \right) = \mathcal{K} \frac{\partial h}{\partial \tau}; \quad (3)
\]
Equations and conditions (1)-(7) contain the following relative values: \( \overline{C} \), \( \overline{S} \) are the concentrations of the small suspended and deposited particles; \( k \) is the hydraulic conductivity; \( \overline{S}_d \) is the deposit concentration (small particles + bound water); \( \overline{V} \) is the filtration rate; \( \lambda \) is the filtration coefficient; \( \tilde{h} \) is the reduced head; \( \gamma \) is the deposit composition coefficient; \( \tilde{I} \) is the thickness of the deposit layer. The surface filtration was considered by analytical methods separately for two main modes due to fundamental differences when solving the corresponding mathematical problems.

2. Surface filtration at a constant rate

Let the filtration rate be kept constant, that is \( V = V_0 = \text{const} \). Then it is easy to find the regularity of the rise of the upper boundary of the dynamic layer with a known porosity \( n_I \), which has in this case a linear form,

\[
l(t) = a_I t, \quad \tilde{l}(t) = \tilde{a}_I t.
\]

where \( a_I = V_0 C_I / (1 - n_I) \), \( \tilde{a}_I = n_I a_I / V_0 \). \( C_I \) is the concentration of the large particles. The calculation equation for \( \overline{S} \) is obtained in the following form

\[
\frac{\overline{S}_I}{\alpha I} \frac{d \tilde{z}}{0 \lambda(\tilde{z})} = \tilde{a}_I \tilde{I} - \overline{\tilde{z}}.
\]

Accordingly, the simple expression is found for the concentration \( \overline{C} \)

\[
\overline{C}(\overline{S}) = 1 - \overline{a}_I \overline{S}.
\]

Therefore, the relative exit concentrations \( \overline{C}_e \) and \( \overline{S}_e \) will change over time as follows

\[
\overline{C}_e(t) = \overline{C}(0, \tilde{r}) = 1 - \overline{a}_I \overline{S}_e(t),
\]

\[
\overline{S}_e(t) = \int_0^\overline{S}_e \frac{d \overline{S}}{0 \lambda(\overline{S})} = \tilde{I}.
\]

Typically, function \( \lambda(S) \) is represented as a series of nonlinear multipliers. Let it be

\[
\lambda(S) = \overline{\lambda}_0(1 + a_{\lambda 1} S)^m_1(1 + a_{\lambda 2} S)^m_2(1 + a_{\lambda 3} S)^m_3,
\]

where \( a_{\lambda i} \), \( m_i \) (\( i = 1, 2, 3 \)) are the empirical constants. Then the relationship between \( \overline{S}_e \) and \( \tilde{I} \) is derived in the following form

\[
(\overline{S}_m - \overline{S}_e) \left[ 1 - \overline{a}_I \overline{S}_e \right]^\omega_1 \left[ (1 + \overline{\theta} \overline{S}_e)^\omega_2 = \overline{S}_m \exp \left[ \frac{\left(1 - \overline{a}_I \overline{S}_m \left(1 + \overline{\theta} \overline{S}_m \right) \tilde{I} \right)}{\overline{a}_I \overline{\theta}} \right].
\]

where \( \omega_1 = 1 - \overline{a}_I \overline{\theta} \), \( \omega_2 = -\overline{a}_I \overline{\theta} \), \( \overline{\theta} \) is the autocatalytic coefficient; \( \overline{S}_m \) is the relative dirt-holding capacity in relation to disperse impurity. The relative head losses were in the dynamic layer

\[
\Delta \tilde{h}(\overline{S}_e(t)) = \overline{a}_I \int_0^{\overline{S}_e(t)} \frac{d \overline{z}}{0 \lambda(\overline{S})} = \overline{S}_e(t).
\]

where the dependence \( \overline{S}_e(t) \) must be established from (12).
Based on (11), it is easy to calculate the relative time \( t_e \) during which the relative exit concentration \( C_e \) decreases to the maximum allowable value \( C_e^* \). Taking into account (10), (11), it is found according to the formula

\[
\tilde{t}_e = \int_0^{1-C_e} \frac{dS}{\lambda(S)(1-\alpha_i S)} \tag{16}
\]

The time \( t_e \) can be adjusted by changing \( \alpha_i \) accordingly.

The suspended substance content steadily decreases with time at the exit of the dynamic layer from the initial maximum concentration \( C_0 \). Its exit concentration will reach the standard value \( C_e^* \) only after a strictly defined time \( t_e \). The required quality of the filtrate is practically guaranteed in the future up to the new filter run.

3. Surface filtration at a constant head drop

By analogy with detachable filtration, it is assumed at a constant head drop that

\[
\bar{x}(S, \nu) = \lambda(S) \nu \tag{17}
\]

This assumption is confirmed by the data of experimental studies [13]. A new relative independent variable is introduced

\[
\tilde{\tau} = \int_0^\tau \nu(\xi)d\xi.
\]

It follows from the balance of the coarsely dispersed impurity in the dynamic layer that

\[
\tilde{t}(\tilde{\tau}) = \tilde{\tau} - \tilde{\tau} \tag{18}
\]

The clarification compartment in the second filtration mode was solved in the same way as in the first mode, and as a result, the dependence for the concentration \( C_e \), which was duplicated (10), was derived, as well as an equation similar in form to (9)

\[
\tilde{\tau} = \tilde{\tau} - \tilde{\tau} \tag{19}
\]

The solution of the hydraulic compartment (3)-(7), first of all, is described by the distribution function of the reduced head

\[
\tilde{\eta}(\tilde{\tau}, \tilde{\xi}) = \nu(\tau) \frac{d\eta}{\int_0^\nu \nu(\nu, \nu)} \tag{20}
\]

To carry out specific calculations, it is necessary to additionally find a relationship between \( \tau \) and \( \tilde{\tau} \). As a result, the dependence of time \( \tilde{\tau} \) on the newly introduced parameter \( S_e^* \) is derived in this form

\[
\tilde{t}(S_e^*) = \frac{1}{\tilde{\tau}_i} \int_0^{\tilde{\tau}_e^*} \frac{dS}{\lambda(S)(1-\tilde{\tau}_i S)} \tag{21}
\]

In this case, the relative filtration rate should also be calculated as a function of \( \tilde{S}_e^* \) according to the formula

\[
\tilde{\nu}(\tilde{S}_e^*) = \left[ \frac{\Delta \tilde{\tau}_0 + \tilde{\tau}(\tilde{S}_e^*)}{\tilde{\tau}_i} \int_0^{\tilde{\tau}_e^*} \frac{dS}{\lambda(S)(1-\tilde{\tau}_i S)} \right]^{-1} \tag{22}
\]

For rational control of the surface filtration in the second main mode, it is also necessary to pre-calculate two operation times. The first time \( t_e \), as in the case of the first mode, is determined based on the maximum permissible impurity concentration at the exit of the dynamic layer \( C_e^* \) according to the following dimensional formula.
The operation time $t_V$ for a decreasing filtration flow rate to its minimum allowable value $V_e$ is established on the basis of formula (22) as $t_h$ in two stages. The ultimate goal of technological calculations is the dimensional value $\Delta t_f$, which is found as follows

$$\Delta t_f = t_V - t_h.$$

4. Calculation of examples and discussion of their results

The quantitative analysis performed on a number of test examples made it possible to visually illustrate the exact solutions obtained above for two mathematical problems of the surface filtration, demonstrate their wide possibilities in predicting the development of the physicochemical situation in a newly formed dynamic layer, and substantiate technological parameters. Typical expressions are accepted for the relative functional filtration coefficient $\lambda(S)$ and hydraulic conductivity $k(S)$. The subject of the predictive and technological calculations for both modes of filtration were the relative values: volume fraction of the small particles of the aqueous suspension (exit value $C_e$), head losses in the entire dynamic layer $\Delta h$, filtration rate $V$, the operation times. The initial parameters from both problems can be conditionally divided into two groups. The values of the parameters from the first group were fixed: $S_m = 200$, $\gamma_c = 0.0035$, $\Delta h_0 = 1$ (the choice of the value $\Delta h_0$ and at the same time $l_0$ is actually not fundamental, since it does not affect the dimensional characteristics of the filtration in any way). The parameters from the second group were alternately changed and thus four examples were compiled. Values $\lambda_0 = 0.02$, $\theta = 0$, $\theta_l = 0.01$ are accepted for the basic example. In the second example, the values $\theta$ and $\theta_l$ remained the same but $\lambda_0 = 0.04$; in the third example, only $\theta$ has changed (0.01), in the fourth example - $\theta_l$ (0.02).

In the first series of calculations, the exit concentration $C_e$ was determined from formula (11) and parameter $S_c$ contained in it was obtained from (12). The set of four curves corresponding to four examples is shown in Fig.1. Of particular note is the sharp decrease in the content of small particles in the filtrate immediately after the start of filtration. Thus, the initial stage of the surface filtration with unsatisfactory filtrate quality should be completed relatively quickly.

![Figure 1](image_url)

Figure 1. Relative volume fraction reduction of the suspended small particles in the filtrate:
1 - basic example, 2 - example 3, 3 - example 2, 4 - example 4
The behavior of function $\Delta h(\tilde{t})$ for both surface and volumetric filtration is generally similar. However, the rate of increase in head losses during the surface filtration turns out to be noticeably lower, first of all, due to the initial breakthrough of a very large number of small particles through a still thin dynamic layer.

A practical application of the solutions obtained above can be the theoretical substantiation of technological parameters, in particular, the content of large particles in suspension. Fig. 2 presents in graphical form the relationship between the relative operation time $\tilde{t}$ and the coefficient of its formation rate $\tilde{a}_r$.

Figure 2: Dependence $\tilde{t}_s(\tilde{a}_r)$: 1 – $\tilde{C}_s = 0.05$, 2 – $\tilde{C}_s = 0.1$, 3 – $\tilde{C}_s = 0.2$

The second series of calculations was again carried out on the first four examples with a fixed value $\tilde{a}_r$ in relation to the second filtration mode. The quality characteristic of the filtrate $\tilde{C}_e$ was determined sequentially in three stages. First of all, with the help of formula (21) the value $\tilde{\tau}$ corresponding to a given time $\tilde{t}$; then, by selection, the value $\tilde{S}_e$ corresponding to the obtained value was also found, and, finally, according to formula (11), the required value $\tilde{C}_e$ was calculated at the time moment. The results of calculations of the content of the small suspended particles at the exit of the surface layer in dynamics are given in Fig. 3.

Figure 3: Decrease in relative filtration rate over time: 1 - example 4, 2 - basic example, 3 - example 3, 4 - example 2

Of fundamental importance for assessing the viability of a dynamic layer in the second filtration mode is the determination of its rate. Its changes with time were directly calculated by formula (22) with the
dependence $S_e(t)$ established from (21). Fig. 4 shows graphs describing the monotonic decay for the data of the same four examples. As an example of a technological calculation for filtration conditions at a constant head drop, the time $t_v$ for reducing its rate to the minimum allowable value $V_\ast$ was set with a continuous variation of the coefficient $\bar{a}_l$. The curves of dependence $\bar{t}_v(\bar{a}_l)$ calculated using equation (22) and formula (21) for different values of $\bar{V}$, are shown in Fig. 4.

![Figure 4: Dependence $\bar{t}_v(\bar{a}_l)$: 1 – $\bar{V} = 0.6$, 2 – $\bar{V} = 0.7$, 3 – $\bar{V} = 0.8$, 4 – $\bar{V} = 0.9$](image)

The conducted technological analysis allows us to assert that an increase in $\bar{a}_l$ that is easily implemented in practice can lead, firstly, to a controlled improvement in the quality of the filtrate and thus achieve a sharp decrease in the impurity content in it, and secondly, to a drop in the filtration rate in the second mode or to a rapid increase in the expenditure of mechanical energy in the first mode.

Thus, surface filtration lets flexible control of the clarification of aqueous suspensions by controlling the formation of the dynamic layer. Therefore, it is increasingly used in the practice of physicochemical water treatment. In the future, it is planned to carry out theoretical studies of the effect of this layer as a secondary membrane in relation to crossflow filtration.

References


