ENHANCED RATE OF HEAT TRANSFER THROUGH PERIODIC TRIANGULAR ARRAY OF CIRCULAR CYLINDERS EMBEDDED IN POROUS MEDIA

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Abstract

Heat transfer enhancement through porous media are of interest in many thermal applications. Forced convection through a triangular array of circular cylinders embedded in a fluid-saturated porous media has been investigated using extended Darcy-Brinkman-Forchheimer momentum equations and Local Thermal Equilibrium (LTE) model. Detailed results are illustrated, for high permeability levels (Darcy number varied from $10^{-3}$ to $10^{-1}$) and different cylinder spacings ($0.7 \leq \phi \leq 0.99$) at $Re = 100$. It is anticipated that porous media would enhance heat transfer, but it emanates multiple order in pressure drop.

Keywords: Triangular array of cylinders, Porous media, Heat transfer enhancement

1 Introduction

Array of cylinders in cross-flow is commonly encountered in many applications such as filtration, polymer, and food processing, and most commonly in heat exchangers\cite{1–4}. Arrangement of cylinders in periodic array is a matter of investigation subject to design and application. A considerable amount of data is required for the design of heat exchangers for choosing the optimal one amongst various design options. Detailing of the most accurate design comes from the experimental results. However, this approach is not only time-consuming but also expensive. For each cylinder’s pattern in an array, a separate model has to be developed which would require those many experimental data. Hence, an optimal design data can be obtained through proper simulation using mathematical models. Consequently, different flow conditions and geometric parameters can easily be analyzed by changing the simulation parameters.

Growing demand for high-efficiency cooling techniques to enhance heat transfer with the least possible frictional losses has brought up the usage of porous media in modern thermal applications for heat transfer enhancement because it renders a large surface area to volume ratio and better mixing in flowing fluid\cite{5}, thereby increasing convective heat transfer.

Heat transfer in fluid-saturated porous media from a single cylinder has been reported in a bunch of literatures\cite{5–10}. Focus has been shifted to study the flow and heat transfer around multiple cylinders in the porous domain concerning the complexity of flow physics and heat transfer. Layeghi\cite{11} numerically investigated the heat transfer around a staggered tube-bank with wooden material (porous) inserts of various conductivities at a fixed Darcy number for $Re=100$ and 300. Numerical results of tube banks with no porous material inserts are compared, and it was found that even ordinary and low conductivity porous material can be used for heat transfer enhancement with proper optimization. Al-Sumaily\cite{12} investigated forced convection case around four cylinders arranged in in-line and staggered pattern immersed in porous media of both metallic or no-metallic type. Using Darcy–Brinkman–Forchheimer's extended model, the influence of cylinders spacing and solid-to-fluid thermal conductivity ratio at different $Re = 1$–250 was examined. Staggered configuration came out to be a better option for thermal performance in comparison with in-line configuration. Wang\cite{13} predicted the effective permeability (porous inserts plus cylinders’ array) by the resultant pressure drop in both square and triangular arrays for different cylinder sizes.

All of the above studies are limited for fixed Darcy numbers and cylinder spacing. Effects of different cylinders’ arrangements and various porous media (particle size, porosity, thermal conductivity ratio, etc.) on the flow and heat transfer in multiple cylinders is still unaccounted in literature. In this article, flow fields (together with possible recirculating wake), rate of enhanced heat transfer and the resistance offered by the periodic array (total pressure drop) at various cylinders spacing are examined at $Re=100$ (Re is based on the cylinder’s diameter).
2 Problem Details and Methodology

Two-dimensional cross-flow around a periodic array of isothermally heated circular cylinders (diameter, D) embedded in fluid saturated porous media is depicted in Fig 1(a). A large number of cylinders is assumed so that the end effects can be overlooked, and periodic boundary conditions at the inlet and outlet are considered. A reduced section as a computational domain, shown by the dashed line in Fig. 1, is taken for simulation. The cylinders are fixed with specified spacing l, calculated for different free volume fractions (φ) available for porous media. φ for the case of triangular array is expressed by Eq. (1)[14] as,

\[ \varphi = 1 - \frac{\pi}{2\sqrt{3}} \left( \frac{l}{D} \right)^2 \]

In the present study, \( l / D \) is varied from 1.5 to 10, so that φ remains in the range of 0.7 to 0.99. From the literature[5,9,10], some assumptions are taken into consideration: Flow is laminar, incompressible and the thermophysical properties of the fluid and porous material are constant with temperature. Porous media is isotropic and homogenous and no heat generation is considered inside porous media. Viscous dissipation and thermal dispersion effects are neglected. The porous media is fully saturated with the flowing fluid which is maintained at \( T_o \) (298K) and the cylinders at \( T_c \) (300K). The thermal conductivity ratio is considered as unity. Local thermal equilibrium model is assumed between the fluid and porous phases because the temperature difference between the two phases is negligible[15,16].

![Figure 1: Cross-section of triangular periodic array embedded in porous media (dashed line represents the computational domain)](image)

The following sets of equations containing continuity, Darcy-Brinkman-Forchheimer extended momentum, and LTE energy equations respectively, are applied [5,17]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\rho}{\varepsilon^2} \left( \frac{\partial^2 u}{\partial t^2} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\mu}{\varepsilon} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu_f}{\kappa} \frac{\rho C_p}{\sqrt{\kappa}} |U| u
\]

\[
\frac{\rho}{\varepsilon^2} \left( \frac{\partial^2 v}{\partial t^2} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\mu}{\varepsilon} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu_f}{\kappa} \frac{\rho C_p}{\sqrt{\kappa}} |U| v
\]
\[( \rho C) (\frac{\partial T}{\partial t} + v (\rho C)) = k_{eff} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  \( (5) \)

where \((u, v)\) are the velocity components, \(T\) is the temperature, \(|U| = \sqrt{u^2 + v^2}\) is the velocity magnitude. \(\rho, \mu_f\) and \(p\) are the density, dynamics viscosity and pressure, respectively. \(C\) and \(k_{eff}\) are the specific heat capacity and effective thermal conductivity defined as \(k_{eff} = e k_f + (1-e) k_s\), and \((\rho C)_f = e (\rho C)_s + (1-e) (\rho C)_f\); where indices \(f\) and \(s\) refer to the fluid and solid phases, respectively.

\(C_p\) is a geometric function called inertial factor expressed as \([13,18]\), \(C_p = 1.75/\sqrt{150\epsilon}\). \(\kappa\) and \(\epsilon\) are permeability and porosity, respectively linked together with particle diameter \((d_p)\) constituting the porous media, and correlated by Carman-Kozeny relation as:

\[\kappa = \frac{1}{180(1-\epsilon)^2} \epsilon^5 d_p^2\]  \( (6) \)

Depending on the variation in particle diameter \((d_p)\), the permeability and porosity can be changed independently, which also changes the topography of the entire porous matrix. Therefore, the non-dimensional size of the particle is kept constant \((d_p/D=0.01)\) throughout the simulation procedure to maintain consistency\([19]\). Moreover, the key control parameters in the present study Reynolds number \((Re)\) and Darcy number \((Da)\) are defined as:

\[Re = \frac{\rho UD}{\mu}\]  \[Da = \frac{\kappa}{D^2} = \frac{1}{180(1-\epsilon)^2} \left( \frac{d_p}{D} \right)^2\]  \( (7) \)

No-slip conditions and constant wall temperatures are applied at the cylinder’s surfaces. Periodic boundary conditions are imposed at the inlet and outlet sections (implying recurrence of the temperature and velocity fields in those selected domain). x-directional flow and normal gradients has been ignored. Local thermal equilibrium (LTE) condition is assumed between the fluid and porous media. After a rigorous analysis of the grid and time-step \((\Delta t)\) independence test, 100 grid points on half of the cylinder surface and, \(\Delta t=0.01\) are chosen for all the simulations. Using the finite volume-based CFD solver ANSYS 18.1, the numerical solution to the current problem was solved along with the boundary conditions. In addition, some custom field functions were used incorporating the Darcy-Brinkman-Forchheimer momentum equations. The least-square method, and the QUICK scheme were used to discretize the gradients and convective terms, respectively. The pressure-linked equations were solved using SIMPLE algorithm. The adopted method for the present problem is thoroughly validated with the literature\([1,5,14,20]\), and reported in our most recent research article\([19]\). The values of the Nusselt number\((Nu)\) remain well within 3% of the values reported by Chen and Wung\([20]\) in their study for a staggered array of tubes in the presence of forced convection, and a maximum deviation up to 3.7% is found from the values reported by Al-Sumaily\([5]\) for a single cylinder implanted in porous media.

3 Results and Discussion

The results of all the simulations are recorded after the initial transients in the flow have diminished. In the present study, the impact of \(Da\) and \(\phi\) on the flow and heat characteristics has been investigated. The pressure distribution \((C_p=(p_s-p)/0.5\rho U^2)\), where \(p\) and \(p_s\) are the pressure in the freestream and on the cylinders’ surface, respectively curves along the surface of the cylinder2 in Fig 2 indicate a strong dependency of \(C_p\) on \(Da\) and \(\phi\). Each plot begins at the rear stagnant point of cylinder2 \((0^\circ)\) and continues until the front stagnation point \((180^\circ)\). It is well known that the use of porous media in heat enhancement is a very viable technique, but it fetches a high pressure drop which is undesirable in many heat applications\([5]\). Cylinders with close proximity in absence of porous media experience high pressure gradient along the surface\([21]\). In the present study, the change in \(C_p\) across the cylinders (from the front stagnation point to the rear stagnation point) increases with decrease in Darcy number. Unlike the bare cylinders (i.e., without porous media), the minimum values of \(C_p\) are not around 0–90°, rather it lies somewhere near the rear stagnation point as a result of the porous media, which prevents the formation of recirculation wakes behind the heated cylinders\([19]\).
In Fig. 3, the trend of mean drag coefficient \( \left( C_D = \frac{F_c}{0.5 \rho U^2 D} \right) \), where \( F_c \) denotes the total drag force on the cylinder in the flow direction) is depicted at a different level of permeability for all \( \phi \). A profound effect of permeability can be seen; as Darcy number is decreased, drag values change substantially and the reason can be attributed to the higher pressure drop and increase in viscous resistance within the boundary layer\cite{19}. Also the mean drag coefficient first reduces with an increase in spacing between the cylinders up to \( \phi = 0.9 \) for highly porous material (see Fig. 2 at \( Da=10^{-1} \)) but this trend gets overturned and the drag values invariably increase with cylinders spacings at low permeability level \( (Da=10^{-3}) \). The variation of pressure and drag coefficient can also be justified with the help of mean velocity profile between cylinder1 and cylinder3 as shown in Fig 4. At high permeability \( (Da=10^{-1}) \), the maximum velocities are found closer to the cylinder3 for both \( \phi = 0.7 \) and 0.9, whereas a flat velocity profile can be observed for \( \phi = 0.99 \). However, some negative profiles are obtained in the backside of the cylinder1, notably for \( \phi = 0.7 \) and 0.9 at \( Da=10^{-1} \) suggesting the presence of recirculation wake. A similar results were presented by Wang\cite{13} for square array of cylinders packed up with porous media.
Figure 4: Effect of the Da on mean velocity profile in the gap (between cylinder1 & cylinder3) for all φ

Time-averaged values of local Nusselt number (NuL) have been plotted in Fig 5 to figure out the relative importance of convective heat transfers over conduction with varying permeability levels. The local Nusselt number can also provide a detailed picture about the temperature field so that the presence of hot zones can be identified. From the plots, it is evident that the local Nusselt number is maximum at the front stagnation point (i.e., θ=180°) for all Da except for φ=0.7, where the maximum lies somewhere near 90°. At Da=10−1, a slight increase in NuL can be observed (at θ=0°), and the reason can be attributed to the presence of the recirculation zone[19].

Figure 5: Local Nusselt number on the surface of cylinder2 for all φ at different permeability levels.

The mean Nusselt number (Nu) to demonstrate the effect of Da for all φ is exhibited in Fig 6. Heat transfer increases with decreasing permeability, i.e., when the number of particles increases in the porous zone, thermal diffusion plays an important role over convection. Heat transfer is higher for φ =0.7 at all permeability levels. It is also observed that Nu is enhanced when φ is increased from 0.9 to 0.99, unlike the bare cylinders[1] (i.e., without porous media). Furthermore, a characteristic parameters, the ratio of average Nusselt number to total pressure drop (r=Nu/ΔPtotal) has been calculated for all φ at two extreme values of Da (10−1 and 10−3), to show the trade-off between heat transfer enhancement and pressure drop. Table 1 shows that the ratio r always has a higher value at Da=10−1 than Da=10−3 for all cylinder spacings.

Table 1: Ratio r for all φ at two extreme values of Da for Re=100.

<table>
<thead>
<tr>
<th>φ</th>
<th>r = Nu / ΔP_{total}</th>
<th>Da=10^{-1}</th>
<th>Da=10^{-3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>154.7</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>77.3</td>
<td>45.6</td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td>93.5</td>
<td>51.3</td>
<td></td>
</tr>
</tbody>
</table>
4 Concluding Remarks

In the present research article, the hydrodynamic and thermal attributes of a triangular array of circular cylinders embedded in porous media in forced convection regimes are investigated. This study suggests the possibility of altering unsteady hydrodynamic and thermal behavior using porous media, which may enhance the heat transfer and suppress the recirculation eddies. However, the pressure drop increases with decreasing permeability. The influence of permeability level on the mean drag and pressure coefficients, gap velocities, and Nusselt number are briefly discussed. From economic and practical standpoints, it is deduced that low $\phi$ (smaller cylinders’ spacing) and high permeability are desirable because at low permeability levels, porous media lends extra flow resistance.

References


