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KINETIC ENERGY DISSIPATION DUE TO A FINITE THICKNESS OF A BLADE TRAILING EDGE

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Abstract

The equation for kinetic energy dissipation due to the finite dimension of a blade trailing edge is derived in the contribution. It is based on balances of mass and momentum fluxes for a compressible fluid in a two-dimensional case where even high flow deflection can occur. An ambiguity of the compressible flow related with subsonic and supersonic solution is represented here by the two quadratic equations which was obtained during the derivation.

Keywords: Linear blade cascade, kinetic energy dissipation, trailing edge.

1 Introduction

In 1993 Denton [1] wrote down a paper focused on the dissipation of kinetic energy in turbomachinery. In appendix 3 of this paper, derivation of kinetic energy dissipation due to the finite dimension of the blade trailing edge was performed. Denton's approach was based on an assumption of incompressible fluid flow with a small deflection angle at the cascade outlet. Transforming his relationship for the pressure coefficient into the kinetic energy dissipation, one can obtain:

$$\zeta_{te} = \frac{\xi_{te}}{1 + \frac{\kappa M_2^2}{2}} = \left\{ \left(\frac{\delta^* + t}{w} \right)^2 + \frac{\theta}{w} - \frac{C_{pb}t}{w} \right\} \frac{1}{1 + \frac{\kappa M_2^2}{2}}, \quad (1)$$

where $\xi_{te} = (\langle p_{01} \rangle - \langle p_{02} \rangle) / \langle q_2 \rangle$ is the dissipation stagnation pressure coefficient, M_2 is outlet Mach number, κ is the specific heats ratio, $w = o + t$ is the sum of the throat dimension o and the trailing edge thickness t , C_{pb} is the pressure coefficient determined from the blade base pressure. Variables δ^* and θ are momentum and displacement boundary layer thicknesses, respectively.

1.1 Motivation of the work

The work is motivated by the non-physical results obtained from Denton method [1] which is based on an assumption of incompressible fluid flow. These results calculated by Eq. (1) for several levels of expansion ratios are depicted in Figure 1. The problem is for negative values of the kinetic energy dissipation for lowest value of R_{te}/o . These results need a deeper revision due to the violation of the second law of thermodynamic. Data suggests

that there is evolution with expansion ratio (here defined by outlet isentropic Mach number $M_{2, is}$). This is shown in Figure 2, where the data are plotted in terms of ξ_{te} and $M_{2, is}$. The kinetic energy dissipation decreases with increasing expansion ratio for all studied cases.

Figure 1: Data evaluated by the Denton's method, dependencies between R_{te}/o and ζ_{te} .

True, there are more factors that can affect the results from the experiments, e.g. the dimension of the pressure tap positioned at the blade trailing edge or the flow separation. However, data presented here were obtained under the nominal flow condition where no separation occurred.

The question is whether this trend is caused by the simplification in the Denton derivation, or are there some another effects that were not considered.

1.2 Aim of the work

The aim of this contribution is to derive a relationship for the kinetic energy dissipation due to the finite dimension of the blade trailing edge for compressible flow, where even high flow deflection at the cascade outlet can occur.

Figure 2: Data evaluated by the Denton method, evolution of ζ_{te} with Mach number $M_{2,is}$.

2 Derivation of the kinetic energy dissipation

Derivation of the kinetic energy dissipation due to the finite dimension of the blade trailing edge is based on balances of mass and momentum fluxes. A model of two-dimensional flow is used. This assumption can be justified by the experimental setup, where thanks to the large aspect ratio, flow at the blade midspan can be considered as two dimensional. The system of equations consists of mass balance¹:

$$\dot{m} = \int_{ss}^{ps} \rho_1(x) u_1(x) dx = w \frac{\cos(\gamma - \beta)}{\cos(\gamma)} \langle \rho_2 \rangle \langle u_2 \rangle, \quad (2)$$

and momentum balances in direction of x :

$$H_x : \int_o^{te} p(s) ds = \langle p_2 \rangle \int_o^{te} ds + w \frac{\cos(\gamma - \beta)}{\cos(\gamma)} \sin(\beta) \langle \rho_2 \rangle \langle u_2 \rangle^2, \quad (3)$$

and y :

$$H_y : \int_{ss}^{ps} p_1(x) dx + t p_b + \int_o^{te} \rho_1(x) u_1^2(x) dx = w \langle p_2 \rangle + w \frac{\cos(\gamma - \beta)}{\cos(\gamma)} \cos(\beta) \langle \rho_2 \rangle \langle u_2 \rangle^2, \quad (4)$$

where ρ_i are densities, u_i are velocities, angles γ and β are defined in Figure 3, p_i denotes pressures, s is the coordinate along the suction surface of the blade. Subscript $i = 1$ stands for parameters in the throat, whereas $i = 2$ for the parameters evaluated in the plane of measurement at the cascade outlet. The outlet parameters are evaluated by the data reduction method described in detail by Flídr and Kladrbský [2]. Meaning of the limits in the integrals are as follows: **ss** suction surface, **ps** pressure side, **o** position in throat, and **te** trailing edge. The balances are performed for the control area depicted in Figure 3. Fluxes through the boundaries $\partial\mathcal{V}_a$ and $\partial\mathcal{V}_b$ are cancelled out due to the periodical boundary conditions. Note that three coordinate systems are used. The first system (X, Y) is connected with the plane where the measurement with a pressure probe is

¹In the following, integral values will be denoted by angle brackets $\langle \bullet \rangle$.

performed. The second one is rotated by an angle γ with respect to the first one. Finally, the last one is the curvilinear system defined by the suction surface of the blade and by the normal of this surface.

Figure 3: Control volume for balances of mass and momentum fluxes.

The integrals introduced above can be calculated as follows:

$$\int_{ss}^{ps} \rho_1(x) u_1(x) dx = \langle \rho_1 \rangle \langle u_1 \rangle (w - t - \delta^*), \quad (5)$$

$$\int_o^{te} p(s) ds = \langle p_{1s} \rangle, \quad (6)$$

$$\int_o^{te} ds = l, \quad (7)$$

$$\int_{ss}^{ps} p_1(x) dx = \langle p_1 \rangle (w - t), \quad (8)$$

$$\int_o^{te} \rho_1(x) u_1^2(x) dx = \langle \rho_1 \rangle \langle u_1 \rangle^2 (w - t - \delta^* - \theta). \quad (9)$$

Using some elementary trigonometric relations, the angle β can be expressed from Eq. (3) as:

$$\beta = \frac{1}{2} \left\{ \arcsin \left[\left(2 \frac{\langle p_{1s} \rangle - \langle p_2 \rangle s}{\langle \rho_2 \rangle \langle u_2 \rangle^2 w} - \tan \gamma \right) \cos \gamma \right] + \gamma \right\}. \quad (10)$$

To simplify the manipulation of equations, some substitutions are introduced during the following derivation. These substitutions are listed in Appendix A. Mentioned the specific one substitution,

Eq. (2) can be rearranged into:

$$\mathcal{B} \langle \rho_1 \rangle \langle u_1 \rangle = \dot{m}. \quad (11)$$

Introducing following symbol:

$$\Pi = \langle p_1 \rangle^{\frac{1}{\kappa}} \langle p_{01} \rangle^{\frac{\kappa-1}{\kappa}}, \quad (12)$$

and provided the gas as perfect, the continuity equation (11) can be transformed into the quadratic form:

$$\Pi^2 - \Pi + \varphi = 0. \quad (13)$$

The transformation of velocity $\langle u_1 \rangle$ into dimensionless velocity $\langle \lambda_1 \rangle = \frac{\langle u_1 \rangle}{\sqrt{\kappa r T_{00}}}$ was used. This transformation holds for the adiabatic boundary conditions. By solving this equation, two roots π_i can be obtained. Coefficient φ is listed in Appendix A. Knowing two roots π_i of Eq. (13), the stagnation pressure $\langle p_{01} \rangle$ can be expressed as:

$$\langle p_{01} \rangle_i^{\frac{\kappa-1}{\kappa}} = \frac{\pi_i}{\langle p_1 \rangle^{\frac{1}{\kappa}}}. \quad (14)$$

The dimensionless velocity can be expressed as a function of pressure $\langle p_1 \rangle$ as:

$$\langle \lambda_1 \rangle_i = \sqrt{\frac{2}{\kappa-1} \left[1 - \left(\frac{\langle p_1 \rangle}{\langle p_{01} \rangle} \right)^{\frac{\kappa-1}{\kappa}} \right]} = \sqrt{\frac{2}{\kappa-1} \left[1 - \frac{\langle p_1 \rangle}{\pi_i} \right]}. \quad (15)$$

Note that two velocities will be obtained.

In the following step, Eq. (4) can be rearranged into:

$$\mathcal{A} \langle p_1 \rangle + \mathcal{D} \langle \lambda_1 \rangle + \mathcal{E} = 0. \quad (16)$$

Substituting Eq. (15) into (16) and after some manipulations, the final expression for the pressure $\langle p_1 \rangle$ reads:

$$\langle p_1 \rangle^2 - \chi \langle p_1 \rangle + \psi = 0, \quad (17)$$

where coefficients χ and ψ are listed in appendix A.

Finally, the correct pressure $\langle p_1 \rangle$ has to be chosen out of four obtained values, based on the pressure measurement on the blade surface. After that choice, mean stagnation pressure is calculated from Eq. (14). Then, the kinetic energy dissipation can be determined as:

$$\zeta_{te} = 1 - \frac{1 - \left(\frac{\langle p_{02} \rangle}{\langle p_2 \rangle} \right)^{\frac{\kappa-1}{\kappa}}}{1 - \left(\frac{\langle p_{01} \rangle}{\langle p_2 \rangle} \right)^{\frac{\kappa-1}{\kappa}}}. \quad (18)$$

3 Conclusion

Derivation of the kinetic energy dissipation due to the finite dimension of blade trailing edge was performed. It was shown that the situation is much more complicated compared to Denton's relationship, where small flow deflection and incompressible fluid were used for calculation. It can be seen in Eq. s (13) and (17), where quadratic equations were obtained. This is caused by the ambiguity related to the compressible flow, where two solutions for subsonic and supersonic flow have to be expected. Since the measurement is performed at the cascade outlet, some approximations of δ^* and θ must be used to obtain their values. This will be the topic for future paper, where this approximation, as well as the experimental results, will be discussed.

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References

- [1] Denton J.: Loss Mechanism in Turbomachines, *ASME J. Turbomach.* vol. 115: (1993) pp. 621-656.
- [2] Flídr E., Kladrubský M.: Preparation, measurement and evaluation of a linear blade cascade, *VZLU Report R-7267*, (2020) - in Czech

A Used substitutions

The following substitutions are used in the presented derivation:

$$\mathcal{A} = (w - \mathfrak{t}), \quad (19)$$

$$\mathcal{B} = (w - \mathfrak{t} - \delta^*), \quad (20)$$

$$\mathcal{C} = (w - \mathfrak{t} - \delta^* - \theta), \quad (21)$$

$$\mathcal{D} = \dot{m} \frac{\mathcal{C}}{\mathcal{B}} a_{00}, \quad (22)$$

$$\mathcal{E} = \mathfrak{t} p_b - \dot{m} \cos \beta \langle u_2 \rangle - w \langle p_2 \rangle, \quad (23)$$

$$\varphi = \frac{(\kappa - 1) \dot{m} r T_{00}}{\mathcal{B}^2}, \quad (24)$$

$$\chi = \frac{2\mathcal{E}}{\mathcal{A}} + \frac{2\mathcal{D}^2}{(\kappa - 1) \pi_i \mathcal{A}^2}, \quad (25)$$

$$\psi = \frac{\mathcal{E}^2}{\mathcal{A}^2} - \frac{2\mathcal{D}^2}{(\kappa - 1) \mathcal{A}^2}. \quad (26)$$

List of symbols

Latin		ξ	stagnation pressure dissipation coefficient
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$	substitutions	π	root of quadratic equation
C	pressure coefficient	Π	defined by Eq. (??)
l	throat to trailing edge length	ρ	density
M	Mach number	φ	coefficient defined in App. A
$\dot{m}$	mass flux	χ	coefficient defined in App. A
o	throat dimension	ψ	coefficient defined in App. A
p	pressure	Index	
s	surface coordinate	00	stagnation parameters at the cascade inlet
$\mathfrak{t}$	trailing edge thickness	01	stagnation parameters in the throat
u	flow velocity	02	stagnation parameters at the cascade outlet
w	sum of throat and trailing edge thickness	1	static parameters in the throat
x, y, z	coordinate system in the throat	2	static parameters at the cascade outlet
X, Y, Z	coordinate system in traversing plane	b	base
Greek		pb	base coefficient
β	Flow deflection angle	ps	pressure side
γ	rotation of coordinate system	te	trailing edge
δ^*	displacement boundary layer thickness	ss	suction side
ζ	kinetic energy dissipation	Symbols	
θ	momentum boundary layer thickness	$\bar{\cdot}$	mean values
κ	ratio of specific heats		
λ	dimensionless velocity		