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ON GRAVITY CURRENTS OVER A POROUS FLOOR IN POWER-LAW CROSS-SECTION CHANNELS

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Abstract

The behavior of an inviscid gravity current which propagates over a horizontal porous floor in containers of power-law cross section (CS) is analysed by shallow-water theory. It is shown that the effect of the porous boundary can be incorporated by means of a parameter λ which represents the ratio of the characteristic time of porous drainage to that of horizontal spread. The interesting cases correspond to small values of λ; otherwise the current has drained before any significant propagation can occur. Typical solutions are presented for various values of the parameters and differences to the non-porous boundary currents are pointed out. The methodology is illustrated for flow in typical power-law CS containers $f(z) = 0.5 + 0.5z^{\alpha}$ where α is positive constant.

Keywords: Gravity currents, shallow-water, porosity.

1 Introduction

Gravity currents (GCs) occur whenever fluid of one density spreads primarily horizontally into fluid of different density. In many natural situations the current flows over an impermeable boundary (see Simpson [3]). However, there are many important situations where gravity currents flow over porous media with a consequent loss of mass from the current. Applications include currents impinging on coastal shelfs and accidental collapse of storage tanks, containing toxic or flammable liquids, surrounded by gravel beds.

There have been a number of previous studies which focused on the flow of GCs propagating over an permeable surface of a rectangular channel. It includes both experimental (Thomas et al [4], Acton [1]) and theoretical (Ungarish & Huppert [6]) contributions. Ungarish & Huppert [6] developed a shallow-water (SW) description of the current which, when integrated numerically, was able to accurately model the evolution of the flow and to predict the main stages of propagation of the current. The theoretical results were supported by the experimental data of Thomas et al [4] (TML).

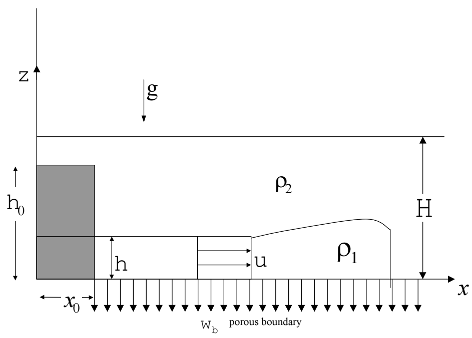

Figure 1: Schematic description of the current released from a lock of length x_0 and height h_0 in the channel of height H with power-law cross-section: Side view.

However, gravity currents generated and spreading in channels with power-law cross-sections are realistic configurations in nature. The examples of such currents include gravity currents propagating in submarine channels, valleys and rivers for which the floor and/or the sides are usually porous and therefore part of the fluid is absorbed in the soil. It is therefore of both practical and academic importance to understand and model the effects of this geometrical property on the flow.

In this work we develop and solve numerically a one-layer shallow-water (SW) model. The present formulation is new and it is an extension of the work of Zemach & Ungarish [7] for permeable containers with power-law forms of cross-sections.

The structure of the paper is as follows. In Sec. 2, the shallow-water model is formulated. We solve the problem numerically and demonstrate the results of finite-difference SW solution for typical power-law cross-sections in Sec. 3. Finally, in Sec. 4 some concluding remarks are given.

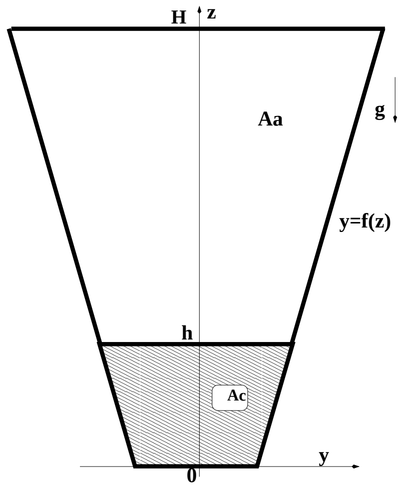

Figure 2: Cross-section of channel. Here $f(z) = f_1(z) + f_2(z)$ is the width of the channel. In the analysis A_a denotes the area occupied by the ambient fluid, A_c is the area occupied by the current, and $A_T = A_c + A_a$ is the total area.

2 Formulation

Consider a gravity current created by release of a fluid of density ρ_1 and kinematic viscosity ν into an ambient fluid of constant density ρ_2 and propagating in a horizontal channel of a uniform in the x-direction cross-section shape and porous horizontal surface at $z = 0$.

The system under consideration is sketched in Figs. 1-2: the bottom and the top of the half-infinite channel are at $z = 0$ and $z = H$ and its side-walls are given by $y = -f_1(z)$ and $y = f_2(z)$ functions. The flow depends actually on the width function, $f(z) = f_1(z)+f_2(z)$, which is assumed continuous and positive and to be in the power-law form $f(z) = 0.5+0.5z^{\alpha}$ (where α is constant). Gravity acts in the $-z$ direction. The current propagates in the positive x-direction. The driving force is the reduced gravity of the current which is defined by

$$g' = (\rho_1 - \rho_2)g/\rho_2, \tag{1}$$

where g is the acceleration due to gravity.

A $\{x, y, z\}$ Cartesian coordinate system with corresponding $\{u, v, w\}$ velocity components is employed. The fluids are assumed to be separated by a sharp, non-entraining, interface which is flat (horizontal) in the y direction.

At time $t = 0$ a given volume of current fluid, initially at rest in reservoir of height h_0 and length x_0, is instantaneously released into the ambient fluid.

One-layer SW model is used: for the ambient fluid domain, where $\rho = \rho_2$, $u = v = w = 0$ is assumed.

The porous boundary is incorporated into the analysis as a boundary condition for the normal velocity component at the bottom, $u_n(x, z = Z_b, t)$, denoted by u_B. This is equal to the rate of discharge through the porous substrate:

$$u_B = -\frac{1}{\tau}(h - Z_b), \tag{2}$$

where $\tau = \nu l/(g'k)$ and l is the thickness of the dense fluid layer in the porous domain; k is a permeability of the porous layer.

The porous boundary contributes to the continuity equation the total flux term at the right hand side. If the bottom of the container is porous, the flux

$$Q_b = -\frac{1}{\tau}hf(0) = -\frac{1}{\tau}\frac{h}{2}. \tag{3}$$

The dimensional variables are used, unless stated otherwise. The position of the interface (the thickness of the current) is $h(x, t)$ and its velocity, averaged over the area of the current, is $u(x, t)$. Initially, at $t = 0$, $h = h_0$ and $u = 0$. We assume a shallow current with $h_0/x_0 \ll 1$. The Reynolds number of the horizontal flow, $Re = h_N u_N/\nu$, where the subscript N denotes value associated with the nose of the current, is assumed to be large.

Let A_c and A_T denote the total cross section of the current and the total of the channel:

$$A_c = A(h) = \int_0^h f(z)dz = \tfrac{1}{2}h(1 + \tfrac{h^\alpha}{\alpha+1}), \qquad A_T = \int_0^H f(z)dz = \tfrac{1}{2}H(1 + \tfrac{H^\alpha}{\alpha+1}). \tag{4}$$

It is convenient to use dimensionless variables defined as follows (here the dimensional variables are denoted by an asterisk):

$$\{x^*, z^*, h^*, H^*, t^*, u^*\} = \{x_0x, h_0z, h_0h, h_0H, Tt, Uu\}, \tag{5}$$

where $U = (h_0g')^{1/2}$ and $T = x_0/U$. The y-direction variables are scaled with the width of the interface in the lock, $f(h_0)$. The scaling of the problem produces the new non-dimensional parameter

$$\lambda = \frac{T}{\tau}, \tag{6}$$

which reflects the ratio between the typical time of propagation of the nose to the typical time of descent of the interface due to the porosity of the boundary, the former over a length x_0 and the latter over a height h_0.

2.1 The governing equations

The inviscid Boussinesq equations of motion are the continuity equation and the momentum equation. In characteristic form this becomes:

$$\begin{pmatrix} h_t \\ u_t \end{pmatrix} + \begin{pmatrix} u & \Psi(h) \\ 1 & u \end{pmatrix}\begin{pmatrix} h_x \\ u_x \end{pmatrix} = \begin{pmatrix} G(h) \\ 0 \end{pmatrix}. \tag{7}$$

Here

$$\Psi(h) = \frac{A(h)}{f(h)} = \frac{h(\alpha + 1 + h^\alpha)}{(\alpha + 1)(1 + h^\alpha)} \tag{8}$$

and

$$G(h) = -\lambda\frac{h}{1 + h^\alpha}. \tag{9}$$

The system (7) is hyperbolic. The eigenvalues of the matrix of coefficients are given by

$$\lambda_\pm = u \pm \sqrt{\Psi(h)}. \tag{10}$$

Consequently, the relationships between the variables on the characteristics are as follows:

$$dh \pm \sqrt{\Psi(h)}du = G(h)dt \quad \text{on} \quad \frac{dx}{dt} = \lambda_\pm. \tag{11}$$

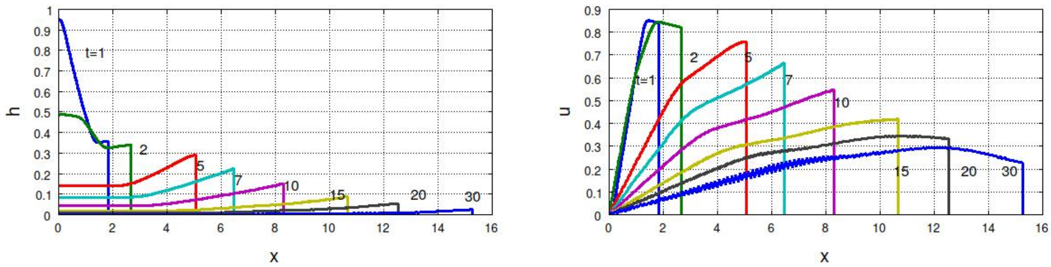

Figure 3: SW model results for trapezoidal configuration with a permeable floor: profiles of h and u as functions of x at various $t = 1, 2, 5, 7, 10, 15, 20, 30$. Here $f(z) = 0.5 + 0.5z, H = 10, \lambda = 0.1$.

The initial and boundary conditions are $u = 0, h = 1$ at $t = 0$ in the lock $0 \leq x \leq 1$ and $u = 0$ at the backwall $x = 0$.

To obtain realistic gravity current solutions, the system must be subjected to a boundary condition at the nose $x = x_N(t)$. The speed boundary condition for the nose, in dimensionless form, is given by

$$u_N = h_N^{1/2} Fr(a), \tag{12}$$

where $a = h_N/H$ and $Fr(a)$ is Froude number function, defined by Ungarish [5]:

$$Fr(a) = Fr(\varphi) = \left[\frac{2(1-\varphi)^2}{1+\varphi}(1+Q)\right]^{1/2}, \tag{13}$$

where

$$\varphi = \frac{A}{A_T} \quad \text{and} \quad Q = \frac{\int_0^h zf(z)dz}{h \cdot [A_T - A]}. \tag{14}$$

3 Finite-difference results

We use a two-step Lax-Wendroff finite-difference efficient method to obtain the $h(x,t), u(x,t)$ and $x_N(t)$ (Morton & Mayers [2]). To facilitate the implementation of the boundary conditions, the x-coordinate was mapped into the coordinate $y = x/x_N(t)$. This transformation keeps the current in the fixed domain $0 \leq y \leq 1$. The SW results displayed here were obtained with, typically, 200 grid points in the $[0, x_N]$ interval, and time step of $1 \cdot 10^{-3}$.

The typical behavior of the time-dependent currents propagating in channels of trapezoidal cross-section ($f(z) = 0.5 + 0.5h$) with permeable floor with $H = 10$ and $\lambda = 0.1$ is shown in Fig. 3.

In the initial stage of the motion the fluid initially collapses from the front to $h_N \approx 0.34$. The porosity of the floor causes a continuous decrease of h_N, as expected. After the adjustment time $t \approx 15$, the current tends to similarity form with the head-up tail-down smooth profile and non-monotonic function of x for the velocity u.

Additional insights were obtained by further comparison between the prediction of distance of propagation x_N with time t. Fig. 4 shows this comparison for floor-permeable containers of various power-law cross-sections. Here $f(z) = 0.5(1 + z^\alpha)$ and $\alpha = 0, 0.5, 1.0, 2.0$, $H = 10$ and $\lambda = 0.1$.

Quiantively, the slowest propagation is achieved in rectangular container ($\alpha = 0$). However, the fastest current is one propagated in container with $\alpha = 0.5$ and it decreases with α. At the initial stages, the current speed of propagation is similar for all cases (till $\approx t = 7$). Then there is a difference between the rectangular and all other cases - the others propagate together till about $t = 15$. Next, in containers with $\alpha = 0.5$ and $\alpha = 1$ the current continue to propagate with insignificant differences till $t \approx 27$, while the spreading in the parabolic ($\alpha = 2$) container becomes a little bit slower.

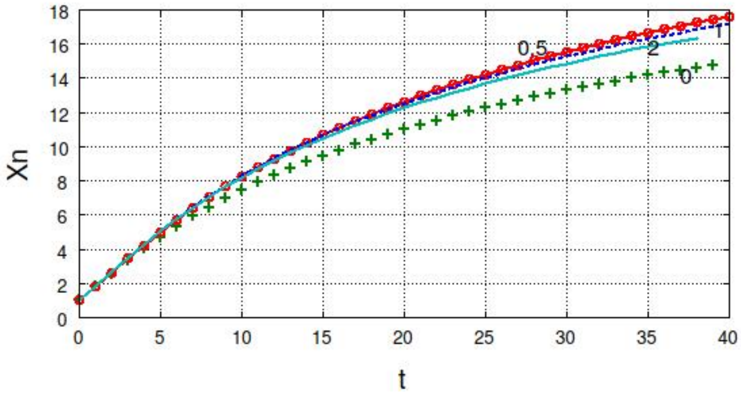

Figure 4: Distance of propagation of the current in containers with permeable floor. Here $H = 10, \lambda = 0.1$. $f(z) = 0.5(1 + z^{\alpha}), \alpha = 0, 0.5, 1, 2$.

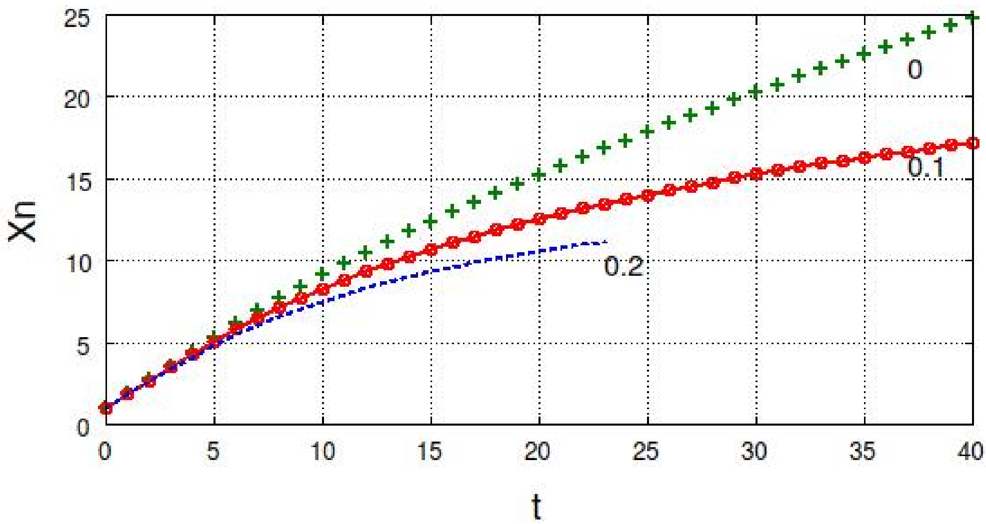

Figure 5: Distance of propagation of the current in the trapezoidal container with permeable floor. Here $H = 10, f(z) = 0.5(1 + z^{\alpha})$ and various values of $\lambda = 0, 0.1, 0.2$.

Fig. 5 gives a quick insight into the influence of the parameter λ on the propagation distance in containers of trapezoidal CS. Initially, the current "does not feel" the permeable floor and propagates till $t \approx 5$ as one over an impermeable boundary. Afterwards, the porosity slows the current down and shortens the effective distance of propagation. These effects become more pronounced as the parameter λ increases. In particular, the permeability of the floor, causes dramatic decreasing of distance of propagation x_N which approaches $\approx 50\%$ at the late times for $\lambda = 0.1$.

Similar behaviour was obtained also in other geometries.

4 Summary and conclusions

In this study we have developed a theoretical model of gravity current flows driven over a porous floor in channels of power-law cross sections described by $f(z) = 0.5 + 0.5z^{\alpha}$ (where $\alpha = const$). A Boussinesq one-layer shallow-water model was used. The effect of the porosity is described by means of a parameter λ which is assumed to be small. To our knowledge, the closed formulation of the general problem developed here is novel and has not been published before. To solve the SW problem, we employed a simple finite-difference code based on a Lax-Wendroff two-step method.

A brief comparison shows that in all tested containers the porosity has little influence on the spreading of the current during an initial stage of propagation, however, afterwards, the porosity slows the current down and shortens the effective distance of propagation. This effect become more pronounced as the parameter λ increases.

Our work also elucidates similarities and differences between the propagation of the current in power-law cross-section and in the classical rectangular case. Quiantively, the slowest propagation is always achieved in rectangular container for any value of λ.

Unfortunately, there are no experimental or Navier-Stokes-simulation data for comparison with the model. The interpretation and implications of this performance requires further investigation. We hope that this paper will motivate for such experiments and Navier-Stokes simulations in the near future.

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