THE HIGH-ACCURACY NUMERICAL SCHEME FOR THE BOUNDARY INTEGRAL EQUATION SOLUTION IN 2D LAGRANGIAN VORTEX METHOD WITH SEMI-ANALYTICAL VORTEX ELEMENTS CONTRIBUTION ACCOUNTING

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Abstract

The problem of development of the high-order numerical scheme for 2D flows simulation in fully Lagrangian meshless vortex methods is considered. The main computation variable is vorticity; its evolution in the flow (transfer and diffusion) is simulated through the motion of vortex elements taking into account the so-called diffusive velocity according to the Viscous Vortex Domains method. Vorticity generation on the airfoil surface line is described by the integral equation, for which solution the original numerical scheme is developed. It deals with piecewise-quadratic vorticity distribution along the airfoil surface line and provides the third order of accuracy. However, in the framework of traditional approach to solution of boundary integral equation, in order to take into account the influence of nearby vortex elements, it is necessary to refine surface line discretization. In practice, it is hardly possible to achieve surface mesh refinement, sufficient for correct computation of the contributions of the vortex elements in the flow, so semi-analytical correction procedure is suggested. All computational formulae are derived.

Keywords: vortex method, boundary integral equation, curvilinear panels, Galerkin approach, high-order scheme, correction procedure, analytical solution.

1 Introduction

Despite the long history of vortex methods, their theory hardly can be considered fully developed. Even in 2D formulation of the problem, the questions connected with existence and uniqueness of the solution, the convergence of the numerical solution to the exact one, the optimal choice of numerical schemes for numerical modeling in various problems, their accuracy and numerical estimates of complexity, at least at the empirical or semi-empirical level, etc, remain actual.

To increase the accuracy of modeling the flow around aerodynamic surfaces using vortex methods, it is necessary to implement more accurate numerical method for solving of the boundary integral equation on its surface with respect to unknown vortex sheet intensity.

In this paper, we consider two-dimensional flows. There are several approaches to airfoil surface line discretization, namely discretization into rectilinear panels and curvilinear ones [1]. In some cases, numerical schemes based on rectilinear (polygonal) approximation of the airfoil surface line, provide rather poor accuracy [2]; it is also impossible to obtain higher than the second order of accuracy. At the same time numerical schemes based on curvilinear panels consideration, allows avoiding such issues. For example, in [3] rather simple test problem is considered of potential flow simulation around elliptical airfoil. It is shown, that higher order numerical schemes usage permits to reduce significantly numerical cost of the algorithm due to coarse surface line discretization.

The example of flow simulation around the circular airfoil at the Reynolds number Re = 1000 by using vortex method is shown in Fig. 1. Positions of discrete vortex elements, simulating the vortex wake, are shown as points, red and blue points denote vortices with positive and negative circulation, respectively. The distance from the airfoil surface line to the closest vortices has order of $10^{-6}$, and in the neighborhood of the airfoil surface line the “density” of the vortex elements is maximal (note, that total number of vortices in the flow has normally order of 1 million). At the same time number of panels, rectilinear or curvilinear, which is necessary to achieve high enough
accuracy in the case of potential flow, is not higher that 1 000, and for curvilinear panel and 3-rd order of accuracy numerical scheme usage can be reduced to approximately 100 or even less.

However, in case of non-potential flow, when there are a lot of vortices close to the airfoil surface line, it is impossible to take into account correctly their influence straightforwardly if the distance from the vortex element to the panel is smaller than the panel’s length. Consequently, in order to solve correctly the described problem for the case shown in Fig. 1 number of panels also should has order of 1 million.

In the present paper the efficient semi-analytic technique is developed, which makes it possible to improve significantly the quality of numerical solution and to achieve extremely high accuracy even for coarse surface discretization. The suggested approach is based on the usage of rather simple exact solution, known from the mirroring technique.

2 Governing equations

The flow of incompressible media in a two-dimensional formulation is described by the incompressibility and Navier — Stokes equations

\[ \nabla \cdot \mathbf{V} = 0, \quad \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = \nu \Delta \mathbf{V} - \frac{\nabla p}{\rho}, \tag{1} \]

where \( \mathbf{V} \) is the flow velocity field; \( p \) is the pressure field; \( \rho = \text{const} \) and \( \nu \) are the density and kinematic viscosity coefficient, respectively.

For simplicity we consider the flow around immovable airfoil in the unbounded flow domain, however all the results can be applied to more general case of arbitrary movable and deformable airfoil or system of airfoils. The boundary conditions are considered to be the following:

1) the no-slip condition:

\[ \mathbf{V}(r, t) = 0, \quad r \in K; \]

2) the perturbation decay conditions on infinity:

\[ \mathbf{V}(r) \rightarrow \mathbf{V}_\infty, \quad p(r) \rightarrow p_\infty, \quad |r| \rightarrow \infty, \]

where \( \mathbf{V}_\infty \) and \( p_\infty \) are the velocity and pressure in the incident flow.

There are some modifications of 2D vortex methods: discrete vortex method [4], random walk method [5], Viscous Vortex Domains method (VVD) [6]. Note, that the last one seems to be the most efficient for engineering applications. The vorticity is considered as the primary computational variable, so the velocity field for incompressible flow can be reconstructed by using the generalized Helmholtz decomposition (GHD) [7], which for considered particular case of immovable airfoil coincides with the Biot — Savart law:

\[ \mathbf{V}(r) = \mathbf{V}_\infty + \int_{K} \frac{\gamma(\xi) \times (r - \xi)}{|r - \xi|^2} d\xi + \int_{S} \frac{\Omega(\xi) \times (r - \xi)}{|r - \xi|^2} dS(\xi). \tag{2} \]

Here, \( \Omega = \Omega k \) is vorticity distribution in the flow domain \( S \), which is considered to be known; \( \gamma(\xi) = \gamma(\xi)k \) is unknown intensity of the vortex sheet on the airfoil surface line \( K \); \( k \) is unit vector orthogonal to the flow plane.

The equation (2), being considered at the airfoil surface line and taking into account the no-slip boundary condition, makes it possible to write down the vector boundary integral equation (BIE)
with respect to unknown vortex sheet intensity $\gamma(\xi)$, $\xi \in K$. It is suggested in [7] to solve it by projecting onto tangent direction, that leads to the 2-nd kind integral equation of Fredholm-type:

$$\int_K \frac{(r - \xi) \cdot n(r)}{2\pi|r - \xi|^2} \gamma(\xi) d\xi - \frac{\gamma(r)}{2} = -\int_S \frac{(r - \xi) \cdot n(r)}{2\pi|r - \xi|^2} \Omega(\xi) dS - V_\infty(r) \cdot \tau(r), \quad r \in K. \quad (3)$$

Here $n(r)$ and $\tau(r)$ are unit outer normal vector and tangent vector, respectively.

Such approach provides much more accurate numerical solution in comparison to the projecting onto normal direction, that is usually used in implementations of vortex methods [13].

The equation (3) has infinite set of solutions, the unique one can be selected by considering the additional condition [4]

$$\int_K \gamma(r) dl_r = \Gamma, \quad (4)$$

where $\Gamma$ is given value of the velocity circulation around the airfoil.

### 3 General approach for solving boundary integral equation

The simplest and most universal way to numerical solution of a boundary integral equation is the Galerkin approach [1, 8]. The surface line is split into $N$ parts, traditionally called “panels”, which endings correspond to arc length parameter values $s_i$, $i = 0, \ldots, N$, where $s_0 = 0, s_N = L$, $L$ is total length of the surface line; the $i$-th panel corresponds to $s$ in range $[s_{i-1}, s_i]$. The basis and projection functions family $\{\phi_i^j(s)\}$, $i = 1, \ldots, N$, $j = 0, 1, 2$, is introduced:

$$\phi_i^0(s) = \begin{cases} 1, & s \in K_i, \\ 0, & s \notin K_i; \end{cases} \quad \phi_i^1(s) = \begin{cases} \frac{s - s(c_i)}{L_i}, & s \in K_i, \\ 0, & s \notin K_i; \end{cases} \quad \phi_i^2(s) = \begin{cases} 4 \left(\frac{s - s(c_i)}{L_i}\right)^2 - \frac{1}{3}, & s \in K_i, \\ 0, & s \notin K_i, \end{cases}$$

where $K_i = [s_{i-1}, s_i]; L_i = s_i - s_{i-1}$ is the length of the $i$-th panel, $s(c_i)$ is the arc length, which corresponds to its center. Note, that the introduced in such a way basis functions are orthogonal.

The approximate solution has the following form:

$$\gamma(r) = \sum_{i=1}^N \left(\gamma_i^0 \phi_i^0(r) + \gamma_i^1 \phi_i^1(r) + \gamma_i^2 \phi_i^2(r)\right) \quad (5)$$

where the coefficients $\gamma_i^j$ can be found from the orthogonality condition of the equation (3) residual to the projection functions:

$$\sum_{j=1}^N \sum_{q=0}^2 \gamma_j^q \int_{s_{i-1}}^{s_i} \phi_i^q(s) ds \int_{s_{i-1}}^{s_i} Q(s, \sigma) \phi_j^q(\sigma) d\sigma - \frac{1}{2} \sum_{q=0}^2 \gamma_j^q \int_{s_{i-1}}^{s_i} \phi_i^q(s) \phi_j^q(s) ds = \int_{s_{i-1}}^{s_i} \phi_i^p(s) f(s) ds, \quad i = 1, \ldots, N, \quad p = 0, 1, 2. \quad (6)$$

The additional condition (4) now has the following form:

$$\sum_{i=1}^N \sum_{p=0}^2 \gamma_i^p \int_{s_{i-1}}^{s_i} \phi_i^p(s) ds = \Gamma. \quad (7)$$

Thus, the initial BIE (3)–(4) is now discretized and represented as linear system (6)–(7), which can be written down in block-matrix form (using regularization procedure similar to [4]):

$$\begin{pmatrix} A_{00}^0 + D_{00}^0 & A_{01}^0 & A_{02}^0 & I \\ A_{10}^0 & A_{11}^0 + D_{11}^0 & A_{12}^0 & O \\ A_{20}^0 & A_{21}^0 & A_{22}^0 + D_{22}^0 & O \\ L^0 & O & O & 0 \end{pmatrix} \begin{pmatrix} \gamma_i^0 \\ \gamma_i^1 \\ \gamma_i^2 \\ R \end{pmatrix} = \begin{pmatrix} b_i^0 \\ b_i^1 \\ b_i^2 \\ \Gamma \end{pmatrix}, \quad (8)$$
where $A^{pq}$ are matrix blocks of $N \times N$ size; $D^{pp}$ are diagonal matrices:

$$A^{ij}_{pq} = \int_{K_i} \phi_i^p(s)ds \left( \int_{K_j} Q(s, \sigma) \phi_j^q(\sigma)d\sigma \right), \quad D^{pp}_{ii} = -\frac{1}{2} \int_{K_i} \phi_i^p(s)\phi_i^p(s)ds; \quad (9)$$

$b^p$ is the right-hand side vector parts; $\gamma^p = (\gamma_1^p, \ldots, \gamma_N^p)^T$ is vector of unknown coefficients, $p = 0, 1, 2$; $I$ and $O$ are the columns/rows consist of units and zeros, respectively; $L^0$ is a raw consists of curvilinear panel lengths; $R$ is regularization variable.

All the coefficients in case of rectilinear panels can be calculated exactly analytically for piecewise-constant and piecewise-linear basis functions [1, 9]. Such schemes provide the first and second orders of accuracy, respectively, but they are suitable in general case only for uniform or close to uniform airfoil surface line discretization. The last issue can be overcome by introducing curvilinear panels; however in this case $A^{ij}_{pq}$ coefficients can not be calculated analytically. In [1, 3, 11] the original technique is developed for their calculation for curvilinear panels through Taylor expansions with respect to the panel length $L_i$; the resulting formulae are given in [3].

Note, that the numerical schemes with only piecewise-constant basis functions (1-st order of accuracy), as well as with piecewise-constant and piecewise-linear basis functions (2-nd order of accuracy) can be constructed from (8) by excluding the corresponding blocks.

4 Right-hand side coefficients of the linear system

The coefficients of the right-hand side of the linear system (8) according to the Galerkin approach, are expressed as the following:

$$b^i = \int_{K_i} \phi_i^p(s)f(s)ds, \quad i = 1, \ldots, N, \quad p = 0, 1, 2.$$

If there is no vorticity in the flow domain, i.e., the potential flow is considered and in the equation (3) $f(s) = -V_\infty \cdot \tau(s) = f_v(s)$, the right-hand side of system (8), which denote as $b_v$, can be calculated by using the following formulae:

$$f(s) = -V_\infty \cdot \tau(s) \approx -\sum_{i=1}^{N_w} \Gamma_w \frac{(r(s) - r_z) \cdot n(s)}{2\pi |r(s) - r_z|^2} = f_v(s) + \tilde{f}_w(s) \quad (10)$$

The term $f_v(s)$ after discretization according to the Galerkin approach leads to additional terms $b_w$ in the right-hand side of the linear system (8), so now we obtain

$$b_v^i = (b_v)_i^p + (b_w)_i^p, \quad (b_w)_i^p = \sum_{z=1}^{N_w} \Gamma_z (q_z)_i^p,$$

where approximate formulae are derived for the coefficients $(q_w)_i^p$:

$$\frac{(q_w)_i^0}{\delta_{iw}} \approx -\sin \delta_{iw} \left( \frac{L_i}{h_{iw}} \right) \frac{1}{24} \left( 2 \sin 3\delta_{iw} - \tau_{h_{iw}} (3 \cos 2\delta_{iw} + \tau_{h_{iw}} \sin \delta_{iw}) + \tau_{h_{iw}}^2 \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^3,$$

$$\frac{(q_w)_i^1}{\delta_{iw}} \approx \frac{1}{12} \left( \sin 2\delta_{iw} - \tau_{h_{iw}} \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^2 + \frac{1}{480} \left( 6 \sin 4\delta_{iw} - \tau_{h_{iw}} (12 \cos 3\delta_{iw} + 3 \tau_{h_{iw}} \sin 2\delta_{iw} - \tau_{h_{iw}}^2 \cos \delta_{iw}) + \tau_{h_{iw}}^3 \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^4,$$

$$\frac{(q_w)_i^2}{\delta_{iw}} \approx \frac{1}{72} \left( 2 \sin 3\delta_{iw} - \tau_{h_{iw}} (3 \cos 2\delta_{iw} + \tau_{h_{iw}} \sin \delta_{iw}) + \tau_{h_{iw}}^2 \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^3.$$
Here $\zeta_i$, $\zeta'_i$ and $\zeta''_i$ denote the airfoil surface line curvature at the center of the $i$-th panel and its first and second derivatives with respect to the arc length; $h_{iw}$ is the length of the vector $h_{iw} = c_i - r_w$, which connects the $w$-th vortex position with the center of the $i$-th panel; $\delta_{iw}$ is the angle between the tangent unit vector $\tau_i$ at the center of the $i$-th panel and the vector $h_{iw}$ (which value is signed and measured in such a way that the positive angle corresponds to a turn form $h_{iw}$ to $\tau_i$ counterclockwise); all the notations are similar to ones introduced in [1].

A vortex element located close to the airfoil boundary has a significant effect on the solution, and it is impossible to represent it correctly by using (5) on the panels, which lie close to the vortex position [3]. In order to overcome this issue the following semi-analytical approach is developed.

It is well-known, that the contribution of the vortex element, placed at the point $r_w$, to the vortex sheet intensity which is formed on the surface line of circular airfoil, can be found as the superposition of the influences of a system of mirrored vortices [12].

The idea of correction procedure is the following: first panels should be chosen, which lie rather close to the vortex; we assume that these panels have indices in range $k_b$, ..., $k_e$. These panels are replaced with the arcs of osculating circles, having curvatures $\zeta_k$, and the above mentioned systems of mirrored vortices is constructed. Then the approximate solution on these panels is expressed as

$$\gamma(s) = \sum_{i=1}^{N} \sum_{q=0}^{2} \gamma_{ij}^q \phi_i^q(s) + \sum_{k=k_b}^{k_e} \gamma_{ik} \phi_i(s) \phi_k(s).$$

Here $\gamma_{ik}(s) = \frac{1}{\pi} \int K_k \phi_i(s) \phi_k(s) \gamma(\sigma) d\sigma$, where $\gamma(\sigma)$ is the angle between the tangent unit vector $\tau_i$ at the center of the $i$-th panel and the vector $h_{iw}$ (which value is signed and measured in such a way that the positive angle corresponds to a turn form $h_{iw}$ to $\tau_i$ counterclockwise); all the notations are similar to ones introduced in [1].

For the panel with indices from the interval $k \in [k_b, k_e]$ we assume that the introduced function $\gamma_{ik}(s)$ provides such contribution to the solution (on the $k$-th panel), that arises due to the influence of the vortex; this means that

$$\sum_{k=k_b}^{k_e} \int K_k \phi_i(s) \phi_k(s) \gamma(\sigma) d\sigma = \frac{1}{\pi} \int K_k \phi_i(s) \gamma_{ik}(s) d\sigma - \frac{1}{2} \int K_k \phi_i(s) \gamma_{ik}(s) d\sigma = \int K_k \phi_i(s) f_w(s) ds, \quad i \in [k_b, k_e], \quad p = 0, 1, 2, \quad (11)$$

where $f_w(s)$ is a function that has the same form (10) except of no summation, because we consider only one vortex.

For the panels with indices from the interval $k \in [k_b, k_e]$ we assume that the introduced function $\gamma_{ik}(s)$ provides such contribution to the solution (on the $k$-th panel), that arises due to the influence of the vortex; this means that

$$\sum_{k=k_b}^{k_e} \int K_k \phi_i(s) \phi_k(s) \gamma(\sigma) d\sigma = \frac{1}{\pi} \int K_k \phi_i(s) \gamma_{ik}(s) d\sigma - \frac{1}{2} \int K_k \phi_i(s) \gamma_{ik}(s) d\sigma = \int K_k \phi_i(s) f_w(s) ds, \quad i \in [k_b, k_e], \quad p = 0, 1, 2. \quad (12)$$

Finally, for the system (11), which correspond to the result of implementing the correction procedure, taking into account the above introduced notations for matrix coefficients $A_{ij}$ and $D_{ij}^{pp}$ and the right-hand side coefficients $(b_{iw})_i^{p}$ we obtain

$$\sum_{j=1}^{N} \sum_{q=0}^{2} A_{ij}^{pq} \gamma_{ij}^q + D_{ii}^{pp} \gamma_{ii}^p = (b_{iw})_i^{p} + (1 - I_{i,k})(b_{iw})_i^{p} + (b_{iw})_i^{p}, \quad i \in [k_b, k_e], \quad p = 0, 1, 2.$$
Here
\[(b_y)^p = - \sum_{k=1}^{b} \int_{K_k} \phi^p_i(s) ds \int_{K_k} Q(s, \sigma) \tilde{\gamma}_k(\sigma) d\sigma, \quad i = 1, \ldots, N, \quad p = 0, 1, 2.\]

Numerical experiments show, that it is necessary to take into account only the coefficients \((b_y)^0\), while \((b_y)^1\) and \((b_y)^2\) can be neglected.

The most difficult is the computation of that term in sum for \((b_y)^0\), that corresponds to the neighboring panels, i.e., when the panels with indices \(i\) and \(k\) has common point. For this case approximate analytical formula can be derived for the corresponding integral computation. Let us consider two neighboring panels, as it is shown in Fig. 2.

We assume, that \(k = i + 1\), i.e., arc length parameters \(s_i = s_{k-1}\); the lengths of \(k\)-th and the \(i\)-th panels are \(L_i\) and \(L_k\), respectively; the curvature of the airfoil surface line at their common point is \(\kappa\), the derivative of the curvature at the common point with respect to arc length is \(\kappa'\).

The vortex element, which influence is taken into account, has circulation \(\Gamma_w\), and it is placed near the airfoil surface, \(k\) is the index of the panel, for which the correction procedure is performed \((k \in [b_0, b_1])\), the arc length parameter value \(s_{k-\frac{1}{2}} = \frac{1}{2}(s_{k-1} + s_k)\) corresponds to its center; the distance from the vortex to the \(k\)-th panel (more precisely, to the osculating circle) is \(d_{w,k}\).

The projection of the vortex element position onto the \(k\)-th panel corresponds to the arc length parameter \(s_{w}\), then its “shift” from the panel center is \(u_{w,k} = s_w - s_{i-\frac{1}{2}}\) (Fig. 2).

We also introduce notations \(\theta^{(1)}_{w,k}, \theta^{(2)}_{w,k}\) and \(\theta^{(3)}_{w,k}\) for the following angles:

\[\theta^{(1)}_{w,k} = \arctan(d_{w,k}, \frac{L_k}{2} - u_{w,k}), \quad \theta^{(2)}_{w,k} = \arctan(d_{w,k}, \frac{L_k}{2} + u_{w,k}), \quad \theta^{(3)}_{w,k} = \pi - \theta^{(1)}_{w,k} - \theta^{(2)}_{w,k},\]

where \(\theta = \arctan(\eta, \xi)\) means angle for which

\[
\sin\theta = \frac{\eta}{\sqrt{\xi^2 + \eta^2}}, \quad \cos\theta = \frac{\xi}{\sqrt{\xi^2 + \eta^2}}.
\]

The approximate formula can be derived for total vorticity, distributed over the \(k\)-th panel due to analytical taking into account the influence of the vortex element:

\[\tilde{\Gamma}_{w,k} = \int_{K_k} \tilde{\gamma}_k(s) ds \approx \frac{\Gamma_w}{2\pi} \left( \kappa_k L_k - 2\theta^{(3)}_{w,k} - \frac{d_{w,k} \kappa_k}{2} \left( \sin 2\theta^{(1)}_{w,k} + \sin 2\theta^{(2)}_{w,k} \right) \right).\]

Now it is possible to write down the approximate formula for the corresponding integrals in expression for the term \((b_y)^0\):

\[\int_{K_i} \phi^0_i(s) ds \int_{K_k} Q(s, \sigma) \tilde{\gamma}_k(\sigma) d\sigma \approx \frac{\kappa_k L_i}{4\pi} \left( \tilde{\Gamma}_{w,k} + \frac{\kappa'}{\kappa_k} \frac{\Gamma_w}{2\pi} \theta^{(3)}_{w,k} L_k \right) \left( 1 + \frac{\kappa'}{\kappa_k} \frac{2L_k - L_i}{L_i + L_k} \right).
\]

Note, that the last formula are valid for both neighboring panels, i.e., not only in case \(k = i + 1\), but also for \(k = i - 1\).
For the non-neighboring panels the corresponding integral can be approximately calculated much easier:
\[
\int_{K_i} \phi_i(s) ds \int_{K_k} Q(s, \sigma) \tilde{\gamma}_k(\sigma) d\sigma \approx \frac{\tilde{\Gamma}_{w,k}}{2\pi} \frac{h_{ik} \cdot n_i}{|h_{ik}|^2} L_i,
\]
where \( h_{ik} = c_i - c_k \), \( c_i \) and \( c_k \) mean the centers of the \( i \)-th and \( k \)-th panels, respectively; \( n_i \) is outer normal unit vector at the center of the \( i \)-th panel.

The additional condition for the unique solution selection (4) remains the same, except of change \( \Gamma \) to \( \Gamma_w \) in the right-hand side:
\[
\Gamma_w = \Gamma - \sum_{k=1}^{k_n} \tilde{\Gamma}_{w,k}.
\]

5 Numerical example

Let us consider elliptical airfoil with 2 : 1 semiaxes ratio, which surface line is discretized into 100 panels of equal length. The vortex element is placed close to the panel no. 3 at the distance of half a panel length (Fig. 3).

![Figure 3: Elliptical airfoil and a vortex element placed close to the panel no. 3](image)

Numerical solution of the boundary integral equation according to the described technique (without performing solution correction) leads to significant error on the panels no. 2, 3, 4 (Fig. 4, a). After correction (Fig. 4, b and c) the numerical solution becomes much closer to the exact one. Moreover, there is practically no qualitative difference comparing piecewise-linear and piecewise-quadratic numerical schemes.

![Figure 4: Numerical solution on the panels no. 1...5 without correction (a) and with correction for piecewise-linear scheme (b) and piecewise-quadratic scheme (c)](image)
6 Conclusion

All the necessary formulae for the integral equation (3) with additional condition (4) discretization are now derived. Their usage allows to provide the third order of accuracy for numerical solution. The scheme is based on the Galerkin approach, and the original algorithm is suggested for correct influence accounting of the vortex elements, placed in the flow domain close to the airfoil.

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References


