CALCULATION OF NONUNIFORM FLUID FLOWS IN A GRAVITY FIELD

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Abstract

Based on the open source software 2D numerical simulations of incompressible stratified fluids flows have been performed. They are characterized by a wide range of values of internal scales that are not found in a homogeneous liquid. Mathematical model is based on the set of differential equations of inhomogeneous multicomponent fluid mechanics. The method allows analyzing in a single formulation the dynamics and fine structure of flow patterns past obstacles in a wide range of flow parameters. The calculations were performed using parallel computational facilities of the web-laboratory UniHUB. The same system of equations and a general numerical algorithm were used for the whole range of the parameters under consideration. The calculation results are in a qualitative agreement with the data from laboratory experiments. Transient flow patterns past obstacles are analyzed, and physical mechanisms are determined, which are responsible for formation of vortices in regions with high density gradients near the edges of an obstacle. For all the velocities of the body motion, the flow field is characterized by a complicated internal structure. In the flow pattern around motionless body dissipative gravity waves are manifested at the edges of the strip. Around the slowly moving body a group of attached waves is formed in opposite phases at the edges of the wedge. Then, the main flow components become vortices, which are formed around the edge of the wedge and manifested downstream in the wake. With further increase in velocity of the body motion, the flow pattern becomes more non-stationary.

Keywords: stratification, wedge, numerical simulation

1 Introduction

Substances dissolved in liquids and particles suspended in gases under the action of gravity of the Earth are unevenly distributed and form a stable stratification. In accordance with the type of state equation of a continuous medium, the density field is determined by temperature, stratifying agent concentration, and pressure profiles. The non-equilibrium medium with the molecular flow of the stratifying components is at rest only if the density gradients are parallel to the direction of the gravity force. Interruption of the molecular flow at impermeable boundaries of arbitrary shape produces specific flows. This phenomenon, called diffusion-induced flows in physics [1]; mountain and valley (katabatic or anabatic) winds [2] in meteorology; and diffuse boundary layer or Ekman layer on an inclined bottom [3] in physical oceanography, has a significant effect on the structure and dynamics of the atmosphere and hydrosphere. Theoretical studies of stratified flows began in the early 1940s [4]. Medium was the incompressible, infinitely deep to escape edge effects on the finite tilted plate, stratification was selected exponential, governing equations were linearized and flow was supposed stationary. The solution was rediscovered by O. Phillips [5], who calculated and experimentally observed paradoxical ascending motion of a heavy dye fluid near the wall moving opposite to the action of the gravity force. Similar solution, was constructed by C. Wunsch [6].

The first short and long times asymptotic solutions describing the development of diffusion induced flows on an infinitely long tilted wall in continuously stratified fluid describe the multiscale flow was described in [7]. Approximate solutions [7] are consistent with regular expansions of the exact solution of transient problem, also describing the flow with an intrinsic multiscale structure [8]. Asymptotic solution describing formation of diffusion induced flows around a horizontal circular cylinder was shown in [9] for a limiting case of radius enlarging to the infinity. It is matched with exact and asymptotic solutions for flows on a tilted plane in [8].
Laboratory experiments were also performed using highly sensitive shadow-imaging devices that showed the presence of beams of dissipative nonstationary gravitational waves at the obstacle poles, besides for the previously observed large vortices [10].

An integral force that is absent on symmetrical obstacles (sphere, cylinder or horizontal plate, etc.), but takes a finit non-zero value on an inclined plate and other bodies asymmetric with respect to the direction of the gravity force is formed along the slope flows. The resulting pressure gradients are large enough and can cause self-movement of free bodies of neutral buoyancy (“diffusion fish” [11]), which plays an important role in the dynamics of the marine environment. First calculations of the flow structure induced by the motionless horizontal wedge was done in [12].

Self-motion of the free neutral buoyancy wedge was first realized in experiments in [13]. However, due to limitation of the optic technique applied in these experiments, adapted to record the movement of the body only, the pattern of the forming flow was not investigated.

The study of the formation mechanisms of forces leading to the self-movement of bodies of different shapes is of practical interest. In the present paper, we study numerically stratified flow structure and dynamics around a horizontal wedge.

2 Mathematical modelling

Unsteady 2-D problem of the evolution of a flow of continuously stratified fluid flow around the horizontally placed wedge is solved in this study by numerical methods. Mathematical modelling of the problem is based on the equation of incompressible stratified fluid mechanics set with undisturbed density distribution \( \rho_0(z) \) given by a stable salinity profile \( S_0(z) \) where the axis \( 0z \) is directed upright. The governing system includes equations of state for density, continuity, Navier-Stokes accounting for the gravity in the Boussinesq approximation, diffusion of stratifying agent as well as no-slip and no-flux boundary conditions on solid boundaries

\[
\rho = \rho_0(1-z/\Lambda + s), \quad \nabla \cdot \mathbf{v} = 0, \quad (1)
\]

\[
\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{1}{\rho_0} \nabla P + \nabla \Delta \mathbf{v} - g, \quad (2)
\]

\[
\frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s = \kappa_s \Delta s + \frac{v_z}{\Lambda}, \quad (3)
\]

\[
\mathbf{v} |_{\Sigma} = 0, \quad s |_{\infty} = 0, \quad v_x |_{\infty} = U, \quad v_z |_{\infty} = 0, \quad (4)
\]

\[
\frac{\partial s}{\partial n} |_{\Sigma} = -\frac{1}{\Lambda} \frac{\partial z}{\partial n} + \frac{\partial s}{\partial n} |_{\infty} = 0. \quad (5)
\]

Here, \( S = S_0(z) + s \) is total salinity including the salt contraction coefficient, \( S_0(z) \) is the stable salinity profile, \( s \) is the salinity perturbation, \( \rho_0 \) is density at zero (neutral buoyancy horizon), \( \mathbf{v} \) is velocity vector, \( P \) is the pressure except for the hydrostatic one, \( \mathbf{v} \) and \( \kappa_s \) are the constant kinematic viscosity and salt diffusion coefficients, \( t \) is time, \( \nabla \) and \( \Delta \) are Del and Laplace operators, respectively, \( g \) is the gravity acceleration, \( \Lambda = -\rho_0 \frac{d\rho_0}{dz} \), \( N = \sqrt{g/\Lambda} \) and \( T_b = 2\pi/N \) are the buoyancy scale, frequency and period, respectively, \( n \) is external normal to the wedge’s surface \( \Sigma \), \( U \) is external flow velocity. The problem was solved in the coordinate system attached to the wedge.

3 The initial condition (motionless wedge)

The initial condition for the stratified medium is steady flow induced by interruption of diffusion flux of stratifying agent on an impermeable surface of the motionless body. Mechanism of flows formation is manifested by pattern of salinity perturbation field (Fig. 1). Positive values of the visualized value are indicated in red, negative ones – in blue. The calculation results of diffusion induced flows on the motionless wedge were in a qualitative agreement with the laboratory ones.
Table 1. Values of input parameters.

<table>
<thead>
<tr>
<th>No</th>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\rho_0$</td>
<td>density on the neutral buoyancy horizon, kg/m$^3$</td>
<td>1020</td>
</tr>
<tr>
<td>2</td>
<td>$g_z$</td>
<td>gravitational acceleration, m/s$^2$</td>
<td>9.8</td>
</tr>
<tr>
<td>3</td>
<td>$\nu$</td>
<td>kinematic viscosity coefficient, m$^2$/s</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>$k_S$</td>
<td>salt diffusion coefficient, m$^2$/s</td>
<td>$1.41 \times 10^{-9}$</td>
</tr>
<tr>
<td>5</td>
<td>$T_b$</td>
<td>buoyancy period, s</td>
<td>6.3</td>
</tr>
<tr>
<td>6</td>
<td>$L$</td>
<td>wedge length, m</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>$h$</td>
<td>height of the wedge base, m</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Thin layers of deficiency and excess of salinity are attached directly to the upper and lower faces of the lateral surfaces of the wedge, respectively, being manifested by the patterns in form of streaks. Thickness of the layers is non-uniform along the surfaces taking values of 0.16 cm near the apex and 0.11 cm near the base of the wedge. Around the corner points of the wedge the additional fine-scale components are formed.

Field of salinity perturbation gradients reflects the complex periodic structure of the diffusion induced flow (Fig. 2). The horizontal extent of the structure does not contradict the experimental data of the refractive index in a lab tank ("color schlieren method" with a horizontal slit and grating) [14]. The overall structure of the image is typical for stratified flows, in which the forces of buoyancy inhibit vertical motion. Inhomogeneity of the vertical flux of the molecular substances caused by impermeable bodies in the fluid interior or slopes of its boundary creates horizontal components of the density gradient that form the flow even in the absence of additional force factors. In formation of flow structure the important role played by edge effects.

Around the corner points of the wedge the additional fine-scale components are formed. Flowing out from sharp edges thin streams forming along each side of the wedge, generate zero frequency internal
waves. High longitudinal gradient perturbations of salinity of the order \( \frac{\partial x}{\partial n} \approx 4 \cdot 10^{-2} \) recorded near corner points of the wedge where the values of the disturbance of salinity are \( 10^{-3} \). With increasing distance from the obstacles perturbation of salinity gradient decreases sharply and reaches values \( 10^{-6} \) at horizontal distances of about 5 cm and vertical of 0.5 cm.

The detailed study of the diffusion induced flows structure and dynamics were carried out numerically e.g. in [15, 16]. Formation of an intensive zone of intensive depression in front of the leading vertex of the wedge is responsible for generation of propulsive mechanism. Asymmetrical complex structure of the flow creates non-zero integral force acting on the body, which can drive its self-motion. The principal mechanism of the phenomena lies in the pressure deficiency which is sufficient for description of the observed displacement of a body (Fig. 3). The minimum pressure values are fixed on the impermeable wall of the wedge. The intensity of the pressure drops sharply with the distance from the wall.

The motion is driven by available potential energy accumulated in non-uniform density profile in the gravity field. This self-motion phenomenon can be observed at natural conditions and in laboratory experiments in form of horizontal extended streaky structures which are formed due to interruption of natural diffusion flux of stratifying agent on the impermeable surface [14]. A variety of schlieren methods (vertical slot – the Foucault knife or Maksutov’s thread, a coloured schlieren with the grating and a horizontal slit) was applied to visualize a pattern of perturbations near the self-propelled wedge on the horizon of neutral buoyancy. The influence of external conditions on the stability of the self-propulsion of the body is shown.

\[ P \times 10^{-4} \text{ Pa} \]

\[ x, \text{ cm} \]

Figure 3: Axial distribution of pressure \((L = 10 \text{ cm}, h = 2 \text{ cm}, T_0 = 6.28 \text{ s}, U = 0)\).

4 Numerical modeling

The formulated problems were analyzed numerically using the finite volume method realized using in-house developed solvers based on the freely distributed open-source package, OpenFOAM [12, 17]. The general Gauss procedure is used to compute volumetric integrals over a control volume. Interpolation of the convective terms of the equations is carried out using TVD schemes. Discretization of the time derivative is implemented using three-point implicit asymmetrical scheme of second order accuracy with backward differencing. With the purpose of conjugate solving of equations for velocity and pressure, the pressure-implicit split-operator (PISO) algorithm with four correctors is used, that is an iterative procedure usually applied for transient problems.

The computational grid is settled to resolve the finest flow elements in high-gradient regions of the flow especially near the surface of an obstacle. Such a condition leads to a significant increase in the total amount of computational cells that makes it inefficient to perform calculations on personal computers. Therefore the decomposition method was used to divide the calculation domain into a number of subdomains to be processed separately by different processor cores. Decomposition of the computational domain for a parallel run is carried out by a simple geometric decomposition in which the domain is split into pieces in certain directions with an equal number of computational cells in each block. Such an approach allows setting a high spatial resolution of the computational domain and studying the problem in a wide range of the basic parameters for a quite reasonable time. The parallel computations were carried out in the web-laboratory UniHUB (www.unihub.ru) and Research Computing Centre “Lomonosov” (www.parallel.ru).

A set of dimensional parameters is presented in Table 1. They are close to the experiments [14] that were carried out at the Institute for Problems of Mechanics of the Russian Academy of Sciences.
5 Calculation results (moving wedge)

The initial complex structure of the medium formed by diffusion-induced flows is changed dramatically with start of a forced body motion even with low velocity (Figure 4, a). Advanced perturbations, rosettes of transient and extended fields of attached internal waves and downstream wake past extreme points of the body are formed in a continuously stratified fluid.

When the velocity of the external flow is comparable in order of magnitude with a characteristic rate of diffusion induced flows, $U_N^s = \sqrt{\kappa s N}$, the structure of the field for a long time it retains elements of the initial field (Fig. 4, a, b). In this case alternating strip beams remain attached to the sharp corners of the obstacles. When the flow velocity increases, these bundles are transported downstream (Fig. 4, c, b). Increasing the velocity causes a proportional increase in the attached internal wavelength in accordance with the theory of linear formula $\lambda = UT_b$. The sources of the internal waves are the wedge corners, generating intense vertical displacement of fluid. A deviation of fluid layers from the original position of neutral buoyancy creates consequently their periodic oscillations. Irregularities of crests and troughs of internal wave shapes reflect complex pattern of interference of growing transient and attached internal waves [18].

When the velocity increasing, introduced perturbations become more and more pronounced and overlap the flow pattern produced by the weak initial diffusion induced flows (Fig. 4, d, e). Observation of sequence of calculated flow patterns shows that the flow around wedge is non-stationary. Vortices, which are formed periodically at edges, move downstream. Besides vortices near the body, fine interfaces and long internal waves are observed. Internal wave pattern is stationary with respect of the body.

Thus, we consider stratified flows as a combination of the flow components (wave and vortex), which are simultaneously present in the flow and interact actively with each other [19]. But, at a certain flow regime, a certain flow component can be dominant over others, and the flow can be manifested in form of a set of layered fine structural elements, a group of internal waves and upstream perturbations, a quasi-stable chain of vortices, etc.

Against the background of the main horizontal fluid motion near the wedge, a complex system of compensated back flows is formed. This is due to the diffusion effects of an inhomogeneous fluid in the gravitational field. Fig. 5 shows profiles of fluid velocity components in various vertical sections near the wedge up and downstream over a wide range of velocities $U = 10^{-5} \sim 10^{-1} \text{ m/s}$.

A significant difference between the flow pattern of a stratified fluid and a homogeneous one is observed in the field of advanced perturbations upstream (Fig. 5, a, b). Near the leading tip of the wedge some small-scale perturbations are formed producing set of fine structures in the flow near the body boundary. When the external flow velocity greatly exceeds the characteristic velocity of diffusion-induced flows ($U >> U_N^s$), effects of stratification are less apparent. Around the corner points of the wedge the additional fine-scale components are formed. A deviation of fluid layers from the original position of neutral buoyancy creates consequently their periodic oscillations (Fig. 5, c-f). At velocities $U > 10^{-2} \text{ m/s}$ vortex perturbations are formed in the wake of a wedge. The symmetry of the flow pattern relative to the neutral buoyancy horizon is broken downstream as you can see from curves 4 and 5 (Fig. 5, g, h).

6 Conclusions

Flow around a wedge is a complex, multiscale, and transient physical process, which requires additional detailed experimental and theoretical study accounting for diffusion effects, thermal conductivity and compressibility of the medium with control of the observability and solvability criteria for all the physical parameters and structural components of the flows under study.

The mathematical model and the numerical implementation method permitting to study simultaneously all the elements of the internally multi-scale stratified currents without additional hypotheses and links are developed.

Patterns of disturbances in the vicinity of the wedge, in particular the salinity gradient field, which is the physical reason of the subsequent formation of a pressure gradient that causes the fluid flow and compensating self-motion of the free body were calculated. The numerically calculated and laboratory observed flow patterns are consistent.
Figure 4: Horizontal component of salinity gradient perturbations $\partial S/\partial x$
when the wedge velocity increasing: (a-e) $U = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}$ m/s
($L = 10$ cm, $h = 2$ cm, $T_p = 6.28$ s, positive values are indicated in red, negative ones – in blue)
Figure 5: The horizontal (a,c,e,g) and vertical (b,d,f,h) velocity components in sections $x = -5$ cm (a,b), $x \approx 0$ cm (c,d) $x \approx 10$ cm (e,f) $x = 15$ cm (g,h) ($L = 10$ cm, $h = 2$ cm, $T = 6.28$ s, curves 1-5 – $U = 10^{-5}$, $10^{-4}$, $10^{-3}$, $10^{-2}$, $10^{-1}$ m/s).
Acknowledgment

The calculations were performed using the service UniHUB (www.unihub.ru) and Research Computing Centre “Lomonosov” (www.parallel.ru).

References


